

CALIFORNIA STATE UNIVERSITY, LONG BEACH
DEPARTMENT OF MATHEMATICS
COMPLEX ANALYSIS COMPREHENSIVE EXAMINATION
FEBRUARY 8, 2025

Do three problems from each part, for a total of six problems.
Circle the problems you want to be graded.

PART A

Problem 1. Compute

$$\int_{-\infty}^{\infty} \frac{x^2}{1 + 2x^2 + x^4} dx.$$

Problem 2. Find the Laurent series for $f(z) = \frac{1}{z(z-1)(z-2)}$ around $z_0 = 1$.

Problem 3.

(1) Compute

$$\int_{\gamma} \frac{z}{(z-1)(z-2)(z-3)} dz$$

where γ is the circle $|z| = 4$.

(2) Compute

$$\int_{\gamma} \frac{z^8}{(z-1)(z-2)\dots(z-10)} dz$$

where γ is the circle $|z| = 11$.

Problem 4. Consider the polynomial $f(z) = z^4 + 5z + 1$. Find out the number of zeros of f in the annulus $\{z \in \mathbb{C} : 1 < |z| < 2\}$.

Problem 5. Find the Residue at each pole for the function $f(z) = \frac{1}{z^2 \sin z}$ and evaluate the integral $\int_{\mathbb{C}} f(z) dz$ where $\mathbb{C}$ is the circle $|z| = 10$ traversed once in the counterclockwise direction.

PART B

Problem 6. Assume f and g are analytic on $\mathbb{C}$, and assume there are infinitely many q such that $|q| \leq 1$ and $f(q) = g(q)$. Prove that $f = g$ everywhere.

Problem 7. Assume f is entire. Prove that if $\lim_{z \rightarrow \infty} f(z) = 0$, then $f(z) = 0$ for any z .

Problem 8.

- (1) State the fundamental theorem of algebra.
- (2) Prove the fundamental theorem of algebra using the Liouville's theorem.

Problem 9. Assume f is analytic on $\mathbb{C}$, and there is $M > 0$ so that $|f(z)| \leq M(1 + \sqrt{|z|})$ for all z . Prove that f is a constant.

Problem 10.

- (1) State and prove Morera's Theorem.
- (2) Give the statement of a claim or theorem whose proof relies on Morera's Theorem. (You need not provide proof.)