CHAPTER FIVE

Regimentation

§ 33. AIMS AND CLAIMS OF REGIMENTATION

Practical temporary departures from ordinary language have recommended themselves at various points in the foregoing chapter. Most of these were fairly representative of the departures that one actually adopts in various pursuits short of symbolic logic, and none was drastic. There were the provisional refinements of vague terms for special limited purposes of law or of almanacs. There was the still more transitory paraphrasing of ambiguous terms, simple or composite, for getting over a sudden block in communication; but generally these moves keep within ordinary usage. There was the resort to variables and parentheses to clear up structural ambiguities; and if these devices have come to dominate mathematical writing, it is largely because mathematical work is so liable to ambiguities of cross-reference and grouping that the easiest plan is to leave the devices hooked up for the duration. There was the 'such that' expedient for ambiguities of scope; this again is seldom called for except in connection with intricate communication, which takes place mostly in mathematics. Finally there was the resort to infinitive phrases to distinguish between those positions that are meant as referential and those that are not. Something of this sort can be needed once in a while to resolve doubts e.g. as to the locus of Tom's historical error or as to what is exercising the commissioner; still its main utility is rather for analytical studies of reference, belief, desire, than for the first-intension use of language in talking of other things.

Opportunistic departure from ordinary language in a narrow sense is part of ordinary linguistic behavior. Some departures, if the need that prompts them persists, may be adhered to, thus becoming ordinary language in the narrow sense; and herein lies one factor in the evolution of language. Others are reserved for use as needed.

In relation to the concerns of this book, those departures have interested us less as general aids to communication than as present aids to understanding the referential work of language and clarifying our conceptual scheme. Now certain such departures have yet a further purpose that is decidedly worth noting: simplification of theory. A striking case is the use of parentheses. To say of parentheses that they resolve ambiguities of grouping gives little notion of their far-reaching importance. They enable us to iterate a few selfsame constructions as much as we please instead of having continually to vary our idioms in order to keep the grouping straight. They enable us thus to minimize our stock of basic functions, or constructions, and the techniques needed in handling them. They enable us to subject long expressions and short ones to a uniform algorithm, and to argue by substitutions of long expressions for short ones, and vice versa, without readjustments of context. But for parentheses or some alternative convention [1] yielding the foregoing benefits, mathematics would not have come far.

Simplification of theory is a central motive likewise of the sweeping artificialities of notation in modern logic. Clearly it would be folly to burden a logical theory with quirks of usage that we can straighten. It is the part of strategy to keep theory simple where we can, and then, when we want to apply the theory to particular sentences of ordinary language, to transform those sentences into a "canonical form" adapted to the theory. If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways. For example, we would have to make express provision for the contrasting scope connotations of 'any' and 'every' (§ 29). Again we would have to incorporate rules on agreement of tense, so as to disallow inference e.g. of

[1] Łukasiewicz has pointed out that the benefits of parentheses can be gained without their aid by adopting a prepositive symbol for each basic construction (in the sense of § 11) and fixing, for each such construction, the number of terms or sentences that it is to admit as immediate components. See Tarski, p. 39.

'George married a widow' from 'George married Mary and Mary is a widow'. By developing our logical theory strictly for sentences in a convenient canonical form we achieve the best division of labor: on the one hand there is theoretical deduction and on the other hand there is the work of paraphrasing ordinary language into the theory. The latter job is the less tidy of the two, but still it will usually present little difficulty to one familiar with the canonical notation. For normally he himself is the one who has uttered, as part of some present job, the sentence of ordinary language concerned; and he can then judge outright whether his ends are served by the paraphrase.

The artificial notation of logic is itself explained, of course, in ordinary language. The explanations amount to the implicit specification of simple mechanical operations whereby any sentence in logical notation can be directly expanded, if not into quite ordinary language, at least into semi-ordinary language. Parentheses and variables may survive such expansion, for they do not always go over into ordinary language by easy routine. Commonly also the result of such mechanical expansion will display an extraordinary cumbersomeness of phrasing and an extraordinary monotony of reiterated elements; but all the vocabulary and constituent grammatical constructions will be ordinary. Hence to paraphrase a sentence of ordinary language into logical symbols is virtually to paraphrase it into a special part still of ordinary or semi-ordinary language; for the shapes of the individual characters are unimportant. So we see that paraphrasing into logical symbols is after all not unlike what we all do every day in paraphrasing sentences to avoid ambiguity. The main difference apart from quantity of change is that the motive in the one case is communication while in the other it is application of logical theory.

In neither case is synonymy to be claimed for the paraphrase. Synonymy, for sentences generally, is not a notion that we can readily make adequate sense of (cf. §§ 12, 14); and even if it were, it would be out of place in these cases. If we paraphrase a sentence to resolve ambiguity, what we seek is not a synonymous sentence, but one that is more informative by dint of resisting some alternative interpretations. Typically, indeed, the paraphrasing of a sentence S of ordinary language into logical symbols will issue in substantial divergences. Often the result S' will be less ambiguous than S, often it will have truth values under circumstances under which S

has none (cf. §§ 37 f.), and often it will even provide explicit references where S uses indicator words (cf. § 47). S' might indeed naturally enough be spoken of as synonymous with the sentence S'' of semi-ordinary language into which S' mechanically expands according to the general explanations of logical symbols; but there is no call to think of S' as synonymous with S. Its relation to S is just that the particular business that the speaker was on that occasion trying to get on with, with help of S among other things, can be managed well enough to suit him by using S' instead of S. We can even let him modify his purposes under the shift, if he pleases.

Hence the importance of taking as the paradigmatic situation that in which the original speaker does his own paraphrasing, as laymen do in their routine dodging of ambiguities. The speaker can be advised in his paraphrasing, and on occasion he can even be enjoined to accept a proposed paraphrase or substitute another or hold his peace; but his choice is the only one that binds him. A foggy appreciation of this point is expressed in saying that there is no dictating another's meaning; but the notion of there being a fixed, explicable, and as yet unexplained meaning in the speaker's mind is gratuitous. The real point is simply that the speaker is the one to judge whether the substitution of S' for S in the present context will forward his present or evolving program of activity to his satisfaction.

On the whole the canonical systems of logical notation are best seen not as complete notations for discourse on special subjects, but as partial notations for discourse on all subjects. There are regimented notations for constructions and for certain of the component terms, but no inventory of allowable terms, nor even a distinction between terms to regard as simple and terms whose structure is to be exhibited in canonical constructions. Embedded in canonical notation in the role of logically simple components there may be terms of ordinary language without limit of verbal complexity. A *maxim of shallow analysis* prevails: *expose no more logical structure than seems useful* for the deduction or other inquiry at hand. In the immortal words of Adolf Meier, where it doesn't itch don't scratch.

On occasion the useful degree of analysis may, conversely, be such as to cut into a simple word of ordinary language, requiring its paraphrase into a composite term in which other terms are compounded with the help of canonical notation. When this happens, the line of analysis adopted will itself commonly depend on what is sought in the inquiry at hand; again there need be no question of the uniquely right analysis, nor of synonymy.

Among the useful steps of paraphrase there are some, of course, that prove pretty regularly to work out all right, whatever plausible purposes the inquiry at hand may have. In them, one may in a non-technical spirit speak fairly enough of synonymy, if the claim is recognized as a vague one and a matter of degree. But in the pattest of paraphrasing one courts confusion and obscurity by imagining some absolute synonymy as goal.

To implement an efficient algorithm of deduction is no more our concern, in these pages, than was the implementation of communication. But the simplification and clarification of logical theory to which a canonical logical notation contributes is not only algorithmic; it is also conceptual. Each reduction that we make in the variety of constituent constructions needed in building the sentences of science is a simplification in the structure of the inclusive conceptual scheme of science. Each elimination of obscure constructions or notions that we manage to achieve, by paraphrase into more lucid elements, is a clarification of the conceptual scheme of science. The same motives that impel scientists to seek ever simpler and clearer theories adequate to the subject matter of their special sciences are motives for simplification and clarification of the broader framework shared by all the sciences. Here the objective is called philosophical, because of the breadth of the framework concerned; but the motivation is the same. The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality. Nor let it be retorted that such constructions are conventional affairs not dictated by reality; for may not the same be said of a physical theory? True, such is the nature of reality that one physical theory will get us around better than another; but similarly for canonical notations.

§ 34. QUANTIFIERS AND OTHER OPERATORS

Where the objective of a canonical notation is economy and clarity of elements, we need only to show how the notation *could* be made to do the work of all the idioms to which we claim it to be adequate; we do not have to use it. Notations of intermediate richness can be less laborious to use, and different forms have advantages for different purposes. So, reassured that we are in no way compromising our freedom, we can be uncompromising in our reductions.

One striking reduction is already suggested by what we saw in § 29: we can hold our indefinite singular terms to subject position. The idea in § 29 was that this be done just when ambiguity of scope threatens; but now we can as well insist on it as a regular condition of a narrowly canonical grammar. We can even standardize their manner of occurrence a bit further, insisting specifically that they occur always followed by a predicate of the form 'is an object x such that $\ldots x \ldots$'. For this is just the position that an indefinite singular term does end up in once we apply the 'such that' procedure of § 29 and then substantivize the 'such that' clause by prefixing 'object' in order to accommodate variables.

Also we can dispense with almost the entire category of indefinite singular terms. To begin with, the need for a distinction between 'any' and 'each' or 'every' is already removed by our recourse to 'such that' (cf. § 29). 'No', as of the indefinite singular terms 'no poem', 'nobody', 'nothing', can be paraphrased by means of 'each' and negation. The essential forms of indefinite singular terms reduce thus to 'every F' and 'some F' (in the sense of 'a certain F'), where 'F' stands for any general term in substantival form. But now for the striking economy: these two classes of indefinite singular terms are in turn dispensable in favor of just the two indefinite singular terms 'everything' and 'something'. For, as noted in the preceding paragraph, 'every F' and 'some F' are needed only in the positions 'Every F is an object x such that $\ldots x \ldots$' and 'Some F is an object x such that $\ldots x \ldots$'; and obviously we can paraphrase these in turn respectively as:

(1) Everything is an object x such that (if x is an F then $\ldots x \ldots$),

(2) Something is an object x such that (x is an F and $\ldots x \ldots$).

Thus all the indefinite singular terms are got down to 'everything' and 'something', and even these two come never to occur except followed by the words 'is an object x [or y, etc.] such that'. Hence we may conveniently subject 'everything' and those ensuing words to condensed symbolization; and correspondingly for 'something'. Usual notations for these respective purposes are '(x)' and '$(\exists x)$', conveniently read 'everything x is such that' and 'something x is such that'. These prefixes are known, for unobvious but traceable reasons, as *quantifiers*, universal and existential.

A small further economy is still possible: of our two surviving indefinite singular terms, 'everything' and 'something', one only is needed. In other words, existential quantifiers can be paraphrased with help of universal ones and vice versa, as is well known: '$(\exists x)(\ldots x \ldots)$' becomes 'not (x) not $(\ldots x \ldots)$' and conversely.

This last reduction is of little moment. The reduction of all indefinite singular terms to the two sorts of quantifiers is more significant, concentrating as it does the whole bewildering phenomenon of indefinite singular terms in two instances, 'everything' and 'something'. And much more significant still was the stage reached already in § 29: the clear isolation of the scopes of indefinite singular terms. I have explained the idea of quantification in stages, separating its several significant aspects; but Frege achieved the whole thing at once, right down to the final reduction to universal quantifiers, in his *Begriffsschrift* (1879), a thin book that may be said to mark the start of mathematical logic.

The indefinite singular terms were built upon general terms. They have now disappeared into quantification. But there remain definite singular terms that are likewise built upon general terms: notably singular descriptions and demonstrative singular terms (§ 21). Now we can assimilate the demonstrative singular terms to singular descriptions, treating 'this (that) apple' as 'the apple here (there)', '*der hiesige (dortige) Apfel*'. This use of the indicator words 'here' and 'there' as general terms attributively adjoined to 'apple' depends on pointing just as did the use of 'this' and 'that': no less and no more. In the case of 'this (that) apple' the question of spatiotemporal extent, left open by the pointing gesture, was conveniently settled by the general term 'apple' (cf. § 21); but the same happens in the general term 'apple here (there)', for it is true only of things of which both components are true.

A further sort of definite singular term which, like the singular description, is built on a general term, is the class-name. In it the general term in its substantival form is followed by '-kind', or pluralized and preceded by 'the class of the'. Another is the attribute-name (cf. § 25), in which the general term in its adjectival form is followed perhaps by '-ness', or '-ity', or in its verbal form is inflected infinitively or gerundively: 'to be a dog', '(the attribute of) being a dog', 'to be human', 'to err', 'to bake pies'. Another is the relation-name, similarly formed: 'nextness', 'superiority', 'giving'.

We can gain some simplicity of structure and also expedite subsequent developments by regimenting these definite singular terms as follows. Consider the singular description, 'the F'. The general term in the role of 'F' here may be simple or composite; in particular it may have the form 'object x such that $\ldots x \ldots$'. Now we may arbitrarily insist on its having the latter form invariably, since 'F' itself can be expanded at will to 'object x such that Fx'. The canonical form for singular description thus becomes:

(3) the object x such that $\ldots x \ldots$.

Similarly the canonical forms for class abstraction (as it is called) and attribute abstraction become:

(4) the class of the objects x such that $\ldots x \ldots$,
(5) to be an object x such that $\ldots x \ldots$.

Relations can be forced somewhat into line thus:

(6) to be objects x and y such that $\ldots x \ldots y \ldots$.

It must be conceded that (6), with its tandem variables 'x' and 'y', strains the 'such that' idiom beyond what we hitherto reckoned on. Moreover (3)–(6) all seem gratuitous inflations of what there had been. But the gain is this: we can now take the further step of treating the whole complex prefixes of (3)–(6) as unitary operators, absorbing the 'such that'. This is just what we did with the complex prefixes of (1) and (2) when we rendered them as simple quantifiers '(x)' and '$(\exists x)$'.

The prefixes 'the object x such that' and 'the class of the objects x such that' have figured prominently among the basic operators of mathematical logic from Frege and Peano onward, commonly in the condensed notations '(ιx)' and '$\hat{x}$'. For the prefixes in (5) and (6) let us use just the unmodified variables themselves, and then enclose the governed sentence distinctively in brackets. Thus (3)–(6) become:

(7) $(\iota x)(\ldots x \ldots)$, $\hat{x}(\ldots x \ldots)$, $x[\ldots x \ldots]$, $xy[\ldots x \ldots y \ldots]$.

In the last two we have abstraction notations for *intensions:* monadic intensions, or attributes, and dyadic intensions, or relations. In the same spirit we might adopt simply the brackets without prefix to express abstraction of medadic (0-adic) intensions, or *propositions*; thus '[Socrates is mortal]' would amount to the words 'that Socrates is mortal', or 'Socrates's being mortal', when these are taken as referring to a proposition. It will be noted that in conformity with modern philosophical practice I am using the term 'proposition' not for a sentence, but for an abstract object which is thought of as designated by a 'that'-clause. Such an object, e.g. that Socrates is mortal, is thought of as related to a sentence, 'Socrates is mortal', in the way that an attribute, e.g. to be a dog or to bake pies, is related to a general term, 'dog', 'bakes pies'. I would be among the last to discourage the question what manner of objects these might be, but it belongs to a critical phase proper to the next chapter.

The four prefixes in (7) are, like the quantifiers, *variable-binding* operators (§ 28). There is the difference only that whereas quantifiers attach to sentences to produce sentences, these four new operators all attach to sentences to produce singular terms. The sentence to which an operator is attached is called the *scope* of the operator. The scope of a quantifier is not quite the scope, in the sense of § 29, of the indefinite singular term 'everything' or 'something' which the quantifier has absorbed; for the scope of an indefinite singular term included the term itself. The scope of a quantifier or other variable-binding operator is rather the clause governed by the 'such that' which the operator has absorbed.

Actually our operators here are somewhat excessive. That of class abstraction is, as already hinted by the initial 'the' of its verbalization in (4), reducible to that of singular description; for we can paraphrase '$\hat{x}(\ldots x \ldots)$' as:

(8) $(\iota y)(x)(x \in y$ if and only if $\ldots x \ldots)$ [1]

where 'ϵ' is short for the relative term 'is a member of'. If we keep '$\hat{x}$' nevertheless, we do so in the spirit in which we keep '$(\exists x)$' despite its reducibility to universal quantification; viz., as a convenient abbreviation.

This method of eliminating class abstraction fails, by the way, for intensional abstraction. We cannot, on the analogy of (8), paraphrase '$x[\ldots x \ldots]$' as:

(9) $(\iota y)(x)(x$ has y if and only if $\ldots x \ldots)$.

[1] This formula needs modification for some forms of class theory. See my *Mathematical Logic*, pp. 131 ff., 155–166, and "On Frege's way out," pp. 153 ff.

That (8) succeeds where (9) fails is due to a difference in identity conditions for classes and attributes. Since classes with the same members are identical, the condition following '$(\imath y)$' in (8) fixes y uniquely. Since on the other hand attributes are not in general supposed to be identical just because the same things have them, the condition following '$(\imath y)$' in (9) cannot in general be depended on to single out any one attribute y.

§ 35. VARIABLES AND REFERENTIAL OPACITY

Variables now having been brought into increased prominence, it will be worth while to consider more expressly their connection with referential opacity. Each of our variable-binding operators came as a condensation of 'such that' and trimmings; and the variable that the operator binds is the variable bound by the absorbed 'such that'. The bearing of opacity on variables is consequently implicit already in what was said in § 31: that there can be no cross-reference from inside an opaque construction to a 'such that' outside. Rephrased for quantification and other variable-binding operations, this says that *no variable inside an opaque construction is bound by an operator outside.* You cannot quantify into an opaque construction.

When 'x' stands inside an opaque construction and '(x)' or '$(\exists x)$' stands outside, the attitude to take is simply that that occurrence of 'x' is then not bound by that occurrence of the quantifier. An example is the last occurrence of 'x' in:

(1) $(\exists x)(x$ is writing '$9 > x$').

This sentence is true when and only when someone is writing '$9 > x$'. Change 'x' to 'y' in its first two occurrences in (1), and the result is still true when and only when someone is writing '$9 > x$'. Change the last 'x' to 'y', and the case is otherwise. The final 'x' of (1) does not refer back to '$(\exists x)$', is not bound by '$(\exists x)$', but does quite other work: it contributes to the quotational name of a three-character open sentence containing specifically the twenty-fourth letter of the alphabet.

The case of:

(2) $(\exists x)$(Tom believes that x denounced Catiline)

is similar in that 'x' is inside and '$(\exists x)$' outside an opaque construction (if we keep to the convention of § 31). So here again we may say that the '$(\exists x)$' fails to bind that occurrence of 'x'. But (2) differs from (1) in that (1) still makes sense while (2) does not.

Of course these make sense:

(3) $(\exists x)$(Tom believes x to have denounced Catiline),
(4) Tom believes that $(\exists x)(x$ denounced Catiline).

But in each of these the 'x' is bound by '$(\exists x)$'. In (3), 'x' and '$(\exists x)$' are both outside the opaque construction; in (4) they are both inside.

Referential position is primarily thought of, naturally enough, as a position for a singular term that names; and the criterion of such position, viz. substitutivity of identity, was accordingly stated in relation to such terms. Derivatively we have been able to speak of variables in referential position, though they do not name; for the positions remain the same however filled. Section 31 opened on a parallel remark. But now it is time to notice also that the substitutivity criterion can be trained upon the variables directly, without prior talk of constants. For, the substitutivity of identity can be stated with variables as a quantified conditional:

(5) $(x)(y)$(if $x = y$ and $\ldots x \ldots$ then $\ldots y \ldots$)

where '$\ldots x \ldots$' stands for the sentence in which 'x' is said to have purely referential position. Special importance attaches to our being able thus to explain referential position without talking of singular terms other than variables, because in § 38 singular terms other than variables will be eliminated. The notion of referential position will remain.

In (5) the substitutivity of identity takes on a somewhat different air from what it has in:

(6) If Tully = Cicero and ... Tully ... then ... Cicero ... .

We easily produce quite natural sentences for the role of '... Tully ...' that violate (6), and hence we see (6) not as a law of identity but merely as a condition of referentiality of the position of 'Tully' in '... Tully ...'. On the other hand (5) does have the air of a law; one feels that any interpretation of '$\ldots x \ldots$' violating (5) would be simply a distortion of the manifest intent of the blanks. Anyway I hope one feels this, for there is good reason to. Since there is no quantifying into an opaque construction, the positions of 'x' and 'y' in '$\ldots x \ldots$' and '$\ldots y \ldots$' must be referential if 'x' and 'y' in those positions are to be bound by the initial '(x)' and '(y)' of (5) at all. Since the notation of (5) manifestly intends the quantifiers to bind 'x' and 'y' in all four shown places, any interpretation of '$\ldots x \ldots$' violating (5) would be a distortion.

Evidently then a more fundamental characterization of referential position than (5), from the point of view of variables, is the point about binding: occurrences of variables have to be referential relative to the scope of the quantifier that binds them. But if in trying to settle whether a position is referential we feel uncertain of our intuitions about quantifiers and what they bind, we can always fall back on (5) or even on substitutivity of identity for constant terms.

Variable-binding operators, as well as variables, were brought into increased prominence in § 34. We have now got a better perspective on referential position, from the point of view of variables. It turns out that we can also add a certain vividness to the treatment of propositional attitudes (§§ 31, 32) by exploiting certain of the operators. For, the verbs of propositional attitude may be viewed as relative terms predicable of objects some of which are propositions, attributes, or relations. Thus 'Tom believes that Cicero denounced Catiline', 'Tom believes Cicero to have denounced Catiline', and 'Tom believes Cicero and Catiline to have been related as denouncer and denounced' (§ 31) become respectively:

(7) Tom believes [Cicero denounced Catiline],
(8) Tom believes Cicero x[x denounced Catiline],
(9) Tom believes Cicero and Catiline xy[x denounced y].

We may for greater clarity rearrange (8) and (9) thus:

(8) Tom believes x[x denounced Catiline] of Cicero,
(9) Tom believes xy[x denounced y] of Cicero and Catiline.

This move is not an adoption of the Frege-Church theory (§ 31). In sentences of propositional attitude I am taking only each whole opaquely enclosed portion as naming an intension. I do not take its component terms and sentences as naming intensions, nor impute shifts of reference. Why I prefer to touch intensional objects so lightly will appear in Chapter VI, when I undertake to get rid of them altogether.

Sentences (7)–(9) have the respective forms '*Fab*', '*Fabc*', '*Fabcd*'. In (7) 'believes' figures as a dyadic relative term, predicated of a man and a proposition. In (8) 'believes' figures as a part of a triadic relative term 'believes of', predicated of a man, an attribute, and a man. In (9) 'believes' figures as a part of a tetradic relative term 'believes of and', predicated of a man, a relation, and two men. Each of the positions represented by '*a*', '*b*', '*c*', and '*d*' here is, as always, purely referential. The opaque constructions, marked in the verbal formulations by the 'that' and 'to' associated with 'believes', are marked uniformly in (7)–(9) by the brackets of intensional abstraction.

This opacity of intensional abstraction is no mere consequence of our reading these constructions into the idioms of propositional attitude. For presumably identity of propositions and of attributes is to be construed in such a way that

$$[\text{the number of the major planets} > 4] \neq [9 > 4] \qquad \text{and}$$

$$x[\text{the number of the major planets} > x] \neq x[9 > x]$$

even though the number of the major planets = 9. This failure of substitutivity of identity shows that the position of '9' is not referential in '$[9 > 4]$' or '$x[9 > x]$'. But it is referential in '$9 > 4$' and '$9 > x$'. So abstraction of propositions and attributes is opaque. That of relations fares similarly.

Or again let '*p*' and '*q*' represent any two true sentences such that $[p] \neq [q]$. Presumably then

$$[\delta p = 1] \neq [\delta q = 1]$$

(cf. § 31), even though $\delta p = \delta q$; so once more propositional abstraction proves opaque. For a parallel argument regarding attribute abstraction, let A and B be coextensive but distinct attributes. (If there were no such, we could forget attributes and talk instead always of classes.) Presumably then

$$x[x \in \hat{y}(y \text{ has } A)] \neq x[x \in \hat{y}(y \text{ has } B)]$$

even though $\hat{y}(y \text{ has } A) = \hat{y}(y \text{ has } B)$.

Note that in the abstraction of attributes the opaque construction takes in the initial '*x*' along with the brackets. Otherwise the initial '*x*' would be an operator outside, and thus powerless to bind an inside variable.

The topic of opacity will be resumed in § 41.

§ 36. TIME. CONFINEMENT OF GENERAL TERMS

Our ordinary language shows a tiresome bias in its treatment of time. Relations of date are exalted grammatically as relations of position, weight, and color are not. This bias is of itself an inelegance, or breach of theoretical simplicity. Moreover, the form that it takes—that of requiring that every verb form show a tense—is peculiarly productive of needless complications, since it demands lip service to time even when time is farthest from our thoughts. Hence in fashioning canonical notations it is usual to drop tense distinctions.

We may conveniently hold to the grammatical present as a form, but treat it as temporally neutral. One does this in mathematics and other highly theoretical branches of science without deliberate convention. Thus from 'Seven of them remained and seven is an odd number' one unhesitatingly infers 'An odd number of them remained', despite the palpable fallacy of the analogous inference from 'George married Mary and Mary is a widow'. One feels the 'is' after 'seven' as timeless, unlike the 'is' after 'Mary', even apart from any artifice of canonical notation.

Where the artifice comes is in taking the present tense as timeless always, and dropping other tenses. This artifice frees us to omit temporal information or, when we please, handle it like spatial information. 'I will not do it again' becomes 'I do not do it after now', where 'do' is taken tenselessly and the future force of 'will' is translated into a phrase 'after now', comparable to 'west of here'. 'I telephoned him but he was sleeping' becomes 'I telephone him then but he is sleeping then', where 'then' refers to some time implicit in the circumstances of the utterance.

This adjustment lays inferences such as the above ones about seven and George conveniently open to logical inspection. The valid one about seven becomes, with present tenses read timelessly, 'Seven of them then remain and seven is an odd number; therefore an odd number of them then remain.' In this form the inference no longer has an invalid analogue about George and Mary, but only a valid one: 'George marries before now Mary and Mary is a widow now; therefore George marries before now (one who is) a widow now.' (Whether to write 'marries before now' as here, or 'then marries' in parallel to the example about seven, is merely a question whether to suppose that the sentences came on the heels of some reference to a specific past occasion. I have supposed so in the one example and not in the other.)

Such rephrasing of tense distorts English, though scarcely in an unfamiliar way; for the treating of time on a par with space is no novelty to natural science. Of the perplexities that are thus lessened, instances outside the domain of logical deduction are not far to seek. One is the problem of Heraclitus (§ 24). Once we put the temporal extent of the river on a par with the spatial extent, we see no more difficulty in stepping into the same river at two times than at two places. Furthermore the river's change of substance, at a given place from time to time, comes to be seen as quite on a par with the river's difference in substance at a given time from place to place; sameness of river is controverted no more on the one count than on the other.

The problem of Heraclitus was already under control in § 24, without help of the alignment of time with space; but intuitively the alignment helps. Similarly for perplexities of personal identity: the space-time view helps one appreciate that there is no reason why my first and fifth decades should not, like my head and feet, count as parts of the same man, however dissimilar. There need be no unchanging kernel to constitute me the same man in both decades, any more than there need be some peculiarly Quinian textural quality common to the protoplasm of my head and feet; though both are possible.[1]

Physical objects, conceived thus four-dimensionally in space-time, are not to be distinguished from events or, in the concrete sense of the term, processes.[2] Each comprises simply the content, however heterogeneous, of some portion of space-time, however disconnected and gerrymandered. What then distinguishes material substances from other physical objects is a detail: if an object is a substance, there are relatively few atoms that lie partly in it (temporally) and partly outside.

[1] Cf. Goodman, *Structure of Appearance*, p. 94.

[2] They are what Strawson (*Individuals*, pp. 56 f.) has dismissed as *process-things*, "not to be identified either with the processes which things undergo *or* with the things which undergo them. . . . I was concerned to investigate . . . the categories we actually possess; and the category of process-things is one we neither have nor need." He supports his distinctions with examples of usage. Given his concern with usage conservation, I expect he is in the right. But our present concern is with canonical deviations.

Zeno's paradoxes, if they can be made initially puzzling, become less so when time is looked upon as spacelike. Typical ones consist essentially in dividing a finite distance into infinitely many parts and arguing that infinite time must be consumed in traversing them all. Seeing time in the image of space helps us appreciate that infinitely many periods of time can just as well add up to a finite period as can a finite distance be divided into infinitely many component distances.

Discussion of Zeno's paradoxes, as of much else, is aided by graphing time against distance. Note then that such graphs are quite literally a treatment of time as spacelike.

Just as forward and backward are distinguishable only relative to an orientation, so, according to Einstein's relativity principle, space and time are distinguishable only relative to a velocity. This discovery leaves no reasonable alternative to treating time as spacelike. But the benefits surveyed above are independent of Einstein's principle.[3]

Tense, then, is to give way to such temporal qualifiers as 'now', 'then', 'before *t*', 'at *t*', 'after *t*', and to these only as needed. These qualifiers may be systematized along economical lines, as follows.

Each specific time or epoch, of say an hour's duration, may be taken as an hour-thick slice of the four-dimensional material world, exhaustive spatially and perpendicular to the time axis. (Whether something is an epoch in this sense will depend on point of view, according to relativity theory, but its existence as an object will not.) We are to think of t as an epoch of any desired duration and any desired position along the time axis.[4] Then, where x is a spatiotemporal object, we can construe 'x at t' as naming the common part of x and t. Thus 'at' is taken as tantamount to the juxtapositive notation illustrated in the singular term 'red wine' (§ 21). Red wine is red at wine.

[3] Einstein's discovery and Minkowski's interpretation of it provided an essential impetus, certainly, to spatiotemporal thinking, which came afterward to dominate philosophical constructions in Whitehead and others. But the idea of paraphrasing tensed sentences into terms of eternal relations of things to times was clear enough before Einstein. See e.g. Russell, *Principles of Mathematics* (1903), p. 471. For further discussion of tense elimination see my *Elementary Logic*, pp. 6 f., 111–115, 155 ff.; Goodman, *Structure of Appearance*, pp. 296 ff.; Reichenbach, pp. 284–298; Taylor; Williams.

[4] The question of an instant, or epoch of no duration, is best set aside now and subsumed under § 52.

We easily extend 'at' to classes. Where z is mankind, z at t may be explained as the class $\hat{y}(\exists x)(y = (x \text{ at } t) \text{ and } x \in z)$ of appropriate man stages.

We can treat the indicator words 'now' and 'then' on a par with 'I' and 'you', as singular terms. Just as the temporary and shifting objects of reference of 'I' and 'you' are people, those of 'now' and 'then' are times or epochs. 'I now' and 'I then' mean 'I at now', 'I at then'; the custom just happens to be to omit the 'at' here, as in 'red wine'.[5]

'Before' can be construed as a relative term predicable of times. Such constructions as 'x is eating y before t' and 'x is eating y after t' then come through thus:

$(\exists u)(u$ is before t and x at u is eating $y)$,

$(\exists u)(t$ is before u and x at u is eating $y)$.

In this example I have used the progressive aspect 'is eating', in preference to 'eats', because what is concerned is the state and not the disposition; contrast 'Tabby eats mice' (§ 28). Temporal qualifications apply to the latter as well, for there may have been a time when Tabby had no taste for mice, and a time may come when she will lose it. Thus we may say 'Tabby now eats mice', 'Tabby at t eats mice', as well as 'Tabby at t is eating mice', but in the one case we report a phase in her evolving pattern of behavior while in the other we report an incident in her behavior.

In the devices of canonical notation thus far before us there is no suggestion for analyzing the terms 'is eating' and 'eats mice', or even 'eats mice' and 'eats fish', into any common elements. Nor will later pages help us much in the matter. For I know of no general analysis of such terms that improves the situation, however unsatisfactory, in which ordinary language leaves them. For

[5] In *Individuals*, p. 216, Strawson argues against viewing 'now' as a singular term. His argument is that 'now' sets no temporal boundaries. One possible answer might be to defend vagueness; another would be to construe the temporal boundaries as those of the shortest utterance of sentential form containing the utterance of 'now' in question. The latter answer is in our present spirit of artificial regimentation, and we must note that the Strawson passage has a different context. I even share, in a way, an ulterior doctrine that he is there engaged in supporting, for I think it is of a piece with my reflections on the primacy of unanalyzed occasion sentences in the theory of radical translation and of infant learning.

special purposes one may well paraphrase a dispositional sentence like 'Tabby eats mice' into a more elaborate sentence built of canonical notations, the progressive form of the verb, and other materials; but the paraphrase may be expected to include details that suit only the case and purposes at hand and afford no general paradigm. For that matter, our analysis in § 35 left us with no suggestion for analyzing the relative terms 'believes', 'believes of', 'believes of and', etc. into any common elements either.

Where canonical notation is cut off, leaving unanalyzed components, will usually vary with one's purposes (§ 33). But what typically remains unanalyzed has the form of a term; indeed a general term, for we shall see how to eliminate singular terms (§ 38). Moreover, this residual general term regularly ends up in predicative position. We have already been witnessing the trend of general terms to predicative position under regimentation of notation. Thus 'I now have a dog' and 'Every dog barks' show the general term 'dog' as part of an indefinite singular term; their paraphrases:

$(\exists x)(x$ is a dog and I now have $x)$,

$(x)($if x is a dog then x barks$)$

give it predicative position. In 'Turtles are reptiles', 'Paul and Elmer are sons of colleagues', 'Buffaloes have dwindled', and 'I now hear lions', six general terms appear in the plural; in the paraphrases:

$(x)($if x is a turtle then x is a reptile$)$,

$(\exists x)(\exists y)($Paul is son of x and Elmer is son of y and x is colleague of $y)$,

$(\exists t)(t$ is before now and $\hat{x}(x$ is a buffalo$)$ now is smaller than $\hat{x}(x$ is a buffalo$)$ at $t)$,[6]

$(\exists x)(x$ is a lion and I now hear x and $(\exists y)(y \neq x$ and y is a lion and I now hear $y))$ (cf. § 24)

all six have predicative position. Occurrences of general terms in singular terms, in the fashion 'the F' and 'to be F', likewise resolve into predicative position: $(\imath x)Fx$, $x[Fx]$.

[6] See the account, two pages back, of a class at t.

Ways in which one general term can occur in another general term were noted in §§ 21 and 22. One was where a (relative) general term was applied to a singular term to obtain a general term of the form 'F of b'. One was where one general term was adjoined attributively to another; thus 'F G', 'red ball'. Now in both cases the composite general term can, in predicative position, be dissolved: '$(F\ G)x$' reduces to 'Fx and Gx', and '$(F$ of $b)x$' to 'Fxb'. The components end up in predicative position. Similarly for other algebraic modes of composition: thus 'F and G' and 'F or G'. The predication '$(F$ or $G)x$' dissolves to 'Fx or Gx', and the predication '$(F$ and $G)x$' dissolves to 'Fx and Gx'.

Such algebraic constructions are in effect cases of the 'such that' form: 'F or b' is 'object x such that Fxb', 'F G' is 'object x such that Fx and Gx', and so on. The observed dissolution of such constructions in predicative position is thus in effect merely the dissolution of 'such that' in predicative position (§ 29). It is notable that the 'such that' construction—or, what comes to the same thing, the relative clause—has no place in the canonical notation. The construction had been crucial, but in § 34 it was absorbed, in its useful functions, into other more special variable-binding operators.

There do remain unreduced ways in which one general term can occur in another. There is the application of an adverb or syncategorematic adjective to a general term, yielding a more complex general term (§§ 21, 22, 27, 28). There is the juxtaposing of substantival general terms, in the often haphazard senses that such juxtaposition may bear (§ 21). For that matter, there are dispositional combinations like 'eats mice'. I do not say the component general terms in such cases reduce to predicative position; the whole residual general term at which our paraphrasing leaves off is what gets predicative position, not its parts. The internal structure of these recalcitrant compounds is, relative to canonical notation, just not structure. Or, if for special purposes such a term is paraphrased with help of the canonical notation in *ad hoc* ways, then its component terms likewise settle into predicative position. In brief the point is that the only canonical position of a general term is predicative position, whatever the term's uncanonical substructure.

This is still not to say that general terms end up without canonical substructure. But the fact is that they do, except when a certain variant line is adopted that will be considered in § 44. Even that line accords general terms no general terms as immediate components, but it does accord some of them sentences as immediate components.

§ 37. NAMES REPARSED

A constant singular term, simple or complex, will seldom be used in purely referential position unless the speaker believes or makes believe that there is something, and just one, that the term designates. For us who know there is no such thing as Pegasus, the sentence 'Pegasus flies' counts perhaps as neither true nor false. (Cf. § 23.)

There are sentences containing 'Pegasus' that we do count true or false. One example is 'Homer believed in Pegasus', to which we shall recur; but here the position may be held not to be referential. Another example is 'Pegasus exists', or 'There is (such a thing as) Pegasus'; and let us consider whether the position of 'Pegasus' there is purely referential. Certainly, if a sentence of the form '. . . exists' is true, and its subject is supplanted by another term that designates the same thing as it does, the result will be true; so by that standard the position is indeed purely referential. Yet oddly so; for there is little evident sense in '$(x)(x$ exists$)$' or $(\exists x)(x$ exists$)$'.

A look at '$(\exists x)(x$ exists$)$' suggests that our embarrassment may be one of riches: that 'exists' has perhaps no independent business in our vocabulary when '$(\exists x)$' is at our disposal. May we not better spell 'Pegasus exists' itself out as '$(\exists y)(y =$ Pegasus$)$'? On this plan '$(x)(x$ exists$)$' and '$(\exists x)(x$ exists$)$' become '$(x)(\exists y)(y = x)$' and '$(\exists x)(\exists y)(y = x)$' and thus trivially true. What we have done here is to construe 'exists' as an ordinary general term, or predicate, but a trivial one: we have taken 'x exists' as '$(\exists y)(y = x)$', which, like '$x = x$', is true of everything. Still this course leaves anomalies. It seems odd, if '$(x)(x$ exists$)$' is true and 'Pegasus' has purely referential position in 'Pegasus exists', for 'Pegasus exists' to be false. This anomaly withstands the proposed expansion of 'exists': it still seems odd, if '$(x)(\exists y)(y = x)$' is true and 'Pegasus' has purely referential position in '$(\exists y)(y =$ Pegasus$)$', for '$(\exists y)(y =$ Pegasus$)$' to be false. Also there is a certain anomaly in that despite the general tendency mentioned at the beginning of this section we want to single out 'Pegasus exists' or '$(\exists y)(y =$ Pegasus$)$' as false, rather than neither true nor false. Finally, independently of all such technicalities, there is just something wrong about admitting that 'Pegasus' can ever have purely referential position in truths or falsehoods; for the intuitive idea behind "purely referential position" was supposed to be that the term is used purely to specify its object, for the rest of the sentence to say something about (§ 30).

Singular terms which, like 'Pegasus', lack their objects thus raise problems; and not only in connection with the concept of purely referential position. The mere occurrence of truth-value gaps, as we may call them—cases where, in Strawson's phrase, the question of truth value does not arise—would add irksome complications to deductive theory if allowed for. We have indeed never been worried that open sentences lack truth values (§ 28), but open sentences are notationally recognizable. A special awkwardness of the truth-value gaps here under consideration is that they cannot be systematically spotted by notational form. Whether 'Pegasus flies' has a truth value is made to depend on whether there is such a thing as Pegasus. Whether a sentence containing 'the author of *Waverley*' has a truth value is made to depend on whether *Waverley* was written by one man or two. Even such truth-value gaps can be admitted and coped with, perhaps best by something like a logic of three truth values. But they remain an irksome complication, as complications are that promise no gain in understanding.

Let it not be supposed that these various perplexities and complications issue merely from a pedantic distinction between what is false and what is neither true nor false. Nothing would be gained by pooling those two categories under the head of the false; for they are distinguished, under whatever names, in that the one category contains the negations of all its members while the other contains the negations of none of its members.

Such, then, are characteristic troubles over singular terms that fail to designate. The original sin is recorded, after a fashion, in § 22. It was the forming of composite singular terms, and the example was 'this apple'. The following impractical sort of reform may therefore suggest itself. We might insist that a singular term (variables aside) never be permitted the form of a single word unless it was learned in that form, like 'mama' and 'water', through the primitive kind of conditioning that antedated the learning of compound singular terms. We might insist that all other singular terms (variables aside) be rendered as compounds, in ways reflecting how they were learned. Then we might devise techniques for meeting possible failures of designation on the part of these visibly structured singular terms, secure meanwhile in the existence of the designata of the simple ones. This sort of approach recalls, if only in caricature, Russell's early philosophy of proper names and descriptions. Anyway it is hopeless, in that everybody has his peculiar history of term-learning and nobody keeps track of it. Moreover there is no evident reason to depend, for improvements of our conceptual apparatus, upon emended re-enactments of genesis. Continued evolution, sped and guided by creative imagination, has served science better.

The following observation will helpfully narrow our problem. Think of 'a' as a singular term, and '$\ldots a \ldots$' as any sentence containing 'a' in purely referential position. By the substitutivity of identity, since the position is purely referential,

(1) $(x)(\text{if } x = a \text{ and } \ldots x \ldots \text{ then } \ldots a \ldots)$.

I shall suppose 'x' foreign to the sentence represented by '$\ldots a \ldots$'. (Otherwise take another letter.) But then (1) is equivalent, by the elementary logic of quantification, to:

(2) If $(\exists x)(x = a \text{ and } \ldots x \ldots)$ then $\ldots a \ldots$.

Conversely, moreover,

(3) If $\ldots a \ldots$ then $(\exists x)(x = a \text{ and } \ldots x \ldots)$;

for, if $\ldots a \ldots$ then $a = a$ and $\ldots a \ldots$. Now (2) and (3) combine to show that '$\ldots a \ldots$' is equivalent to '$(\exists x)(x = a$ and $\ldots x \ldots)$', which contains 'a' only in the position '$= a$'.[1]

This shows that the purely referential occurrences of singular terms other than variables can be got down to the type '$= a$'. It does not show the same for variables, for see the manifold occurrences of 'x' needed in '$(\exists x)(x = a$ and $\ldots x \ldots)$' itself; but no matter, since the singular terms that raised problems were not the variables.

Now an interesting thing about our being able to manoeuvre those singular terms into a standard position '$= a$' is that '$= a$' taken as a whole is in effect a predicate, or general term; and general terms raise none of the problems that singular terms raise. What suggests itself is that '= Pegasus', '= mama', '= Socrates', etc. be

[1] I may mention in passing, what is familiar to logic students, that this transformation is not unique. Often there is a choice between fixing upon longer or shorter segments of text for the role of '$\ldots a \ldots$'.

parsed anew as indissoluble general terms, no separate recognition of singular terms 'Pegasus', 'mama', 'Socrates', etc. being needed for other positions.

The equation '$x = a$' is reparsed in effect as a predication '$x =a$' where '$=a$' is the verb, the 'F' of 'Fx'. Or look at it as follows. What was in words 'x is Socrates' and in symbols '$x =$ Socrates' is now in words still 'x is Socrates', but the 'is' ceases to be treated as a separate relative term '$=$'. The 'is' is now treated as a copula which, as in 'is mortal' and 'is a man', serves merely to give a general term the form of a verb and so suit it to predicative position. 'Socrates' becomes a general term that is true of just one object, but general in being treated henceforward as grammatically admissible in predicative position and not in positions suitable for variables. It comes to play the role of the 'F' of 'Fa' and ceases to play that of the 'a'.

This reparsing depended on a theorem of confinability of singular terms to the position '$= a$'. But the theorem applied only to purely referential uses of the terms. What of their use, so hard to classify and so beset with anomalies, before 'exists'? It turns out to perfection. Our ill-starred previous suggestion of '$(\exists x)(x =$ Pegasus)', as a paraphrase of 'Pegasus exists', comes into its own when '$x =$ Pegasus' is reparsed as 'x is Pegasus' with 'Pegasus' as general term. 'Pegasus exists' becomes '$(\exists x)(x$ is Pegasus)', and straightforwardly false; 'Socrates exists' becomes '$(\exists x)(x$ is Socrates)', with 'Socrates' as general term, and probably true (with timeless 'is', of course). 'Socrates' is now a general term, though true of, as it happens, just one object; 'Pegasus' is now a general term which, like 'centaur', is true of no objects. The position of 'Pegasus' and 'Socrates' in '$(\exists x)(x$ is Pegasus)' and '$(\exists x)(x$ is Socrates)' is now certainly inaccessible to variables and certainly no purely referential position, but only because it is simply no position for a singular term; 'x is Pegasus' and 'x is Socrates' now have the form of 'x is round'.

There remain unaccommodated those not purely referential uses of singular terms that had other forms than 'a exists'; thus perhaps 'Homer believed in Pegasus'. This example may be expanded to show a sentence within a sentence, thus 'Homer believed that Pegasus exists' or 'Homer believed [Pegasus exists]', the contained sentence being of a form already dealt with. There are other examples that cannot so obviously be accommodated under the propositional attitudes; thus 'Tom is thinking of Pegasus', 'is imagining Pegasus', 'is describing Pegasus', 'is drawing Pegasus'.[2] But perhaps they can with some torturing. Perhaps 'Tom is drawing Pegasus' can be managed somewhat in the fashion 'Tom is making a sketch which he imagines to resemble Pegasus', i.e.:

$(\exists y)$(Tom now is making y and Tom now is imagining $x[x$ resembles Pegasus$]$ of y).[3]

Perhaps 'Tom is imagining Pegasus' can be managed as 'Tom is imagining himself to see Pegasus', i.e.:

Tom now is imagining $x[x$ is seeing Pegasus$]$ of Tom.

The point of such efforts would be to give the singular term referential position with respect to its immediately containing sentence, and so render it amenable to reparsing within that immediate context, opaque though the wider context be.[4]

The proposed reparsing of singular terms as general terms should, on pain of introducing new problems of analysis of general terms, be limited to those singular terms that have no internal structure we care to perpetuate. Which terms these are is a question not of how the terms were first learned, nor of whether they are single English words, but of the particular needs of the argument or investigation that we may imagine ourselves engaged in. The singular terms, other than variables, that are treated as simple in this sense might suggestively be called *names*—namehood then being purely relative to varying projects in hand.[5] The proposed reparsing is then a reparsing of names as general terms.

[2] Cf. Chisholm, "Sentences about believing."

[3] Cf. (8) of § 35. To my treatment of the present example, one is tempted to object that the imagined resemblance on the part of y is not as of now, while y is still in the making; but the answer is that there is no 'now' on the last 'y'.

[4] If the opaque wider context is a quotation, any reparsing within it would indeed be unallowable. But we can suppose quotation antecedently dissolved into spelling; cf. § 30.

[5] But note that this use of 'name' is akin to the use in grammar of 'proper name'. In some writings I have used 'name' rather in the sense of 'that which names'—an extragrammatical sense implying existence of a named object. Hochberg, "The ontological operator," pp. 253 f., wrongly assumes that I equate the latter or referential sense of namehood with the grammatical one.

§ 38. CONCILIATORY REMARKS. ELIMINATION OF SINGULAR TERMS

We can encourage a feeling for the reparsing by letting the epithet 'name' accompany 'Socrates' and the like into their new estate, thus saying that the category of names is not dissipated but only reconstrued as subordinate to that of general terms instead of to that of singular terms. In thus construing names as general terms, what we deviate from is only in part usage, and largely attitude toward usage: the policy of parsing names on a par with singular pronouns and indefinite singular terms. That attitude was even somewhat artificial, for it meant construing the natural 'is' sometimes as copula and sometimes as '$=$'. Nor was it the invariable attitude of logicians in past centuries; they commonly treated a name such as 'Socrates' rather on a par logically with 'mortal' and 'man', and as differing from these latter just in being true of fewer objects, viz. one. Again Leśniewski (1930) is best construed as assimilating names to general terms, though he does not so phrase the matter.[1] Ryle took a step in the same direction in 1933, when, talking specifically of the context 'x exists', he urged that "the term 'x' which from the grammar seems to be designating a subject of attributes, is really signifying an attribute."[2] In § 20, we in turn felt that one might naturally wonder if the distinction between general and singular terms were not overrated. Our assimilation of names into the category of general terms is a partial restoration of that in some ways more natural point of view.

Any question of a distinction between singular and general terms is irrelevant to stimulus synonymy (cf. § 12). Furthermore it is irrelevant to the infantile phase at which such terms as 'mama' are learned (§ 19). Some arbitrariness persists when we apply the distinction to mass terms (cf. § 20). And then there is the arbitrariness of deciding when to treat 'is' as '$=$' and when as copula. All in all, who shall say whether English is more radically modified by a canonical notation in which names consort with the singular pronouns and indefinite singular terms or by one in which they consort with the general terms?

[1] See Leśniewski or Lejewski.

[2] Ryle, "Imaginary objects." In writing "The analysis, which seems to me to be the correct one, is this," he goes farther than I, suggesting that there is but one right analysis.

A conspicuous way in which our reparsing of names as general terms deviates from ordinary usage, as distinct from ordinary categorizations of usage, is in its closing of truth-value gaps. But this was a purpose of the reparsing. It would have been wrong if paraphrase carried a synonymy claim; but it does not (§ 33). A paraphrase into a canonical notation is good insofar as it tends to meet needs for which the original might be wanted. If the form of paraphrase happens incidentally to produce sense where the original suffered a truth-value gap and so was wanted for no purpose, we may just let the added cases turn out as they will. (Example: 'Pegasus flies', originally neither true nor false, now gets paraphrased as '$(\exists x)(x$ is Pegasus and x flies)' and hence becomes false.) Such waste cases, what the computing-machine engineers call don't-cares, are a frequent feature of good paraphrases, as we shall have further occasion to note.

There is a feeling that in reparsing the names as general terms we forfeit part of their meaning, viz. the purport of uniqueness.[3] The notion is that 'Socrates' as general term would be true of one and only one thing just as a matter of contingent fact, whereas the uniqueness of designation of 'Socrates' as singular term is claimed in the very character of the word. This intuitive appeal to meaning may even be conceded to be somewhat intelligible (cf. §§ 12, 14), whatever its cogency. But remember that general terms frequently obey laws that seem accountable to the meanings of the terms and not to contingent fact; witness the law of symmetry of the relative term 'cousin', or of transitivity of 'part'. One might equally well recognize uniqueness—anyway in the weak sense "one at most"—as similarly implicit in the very meaning of certain general terms, viz. ones like 'Socrates'. Such general terms might on that very account be denominated, more particularly, names.

Terms like 'Socrates' do ordinarily purport uniqueness of reference not only in the weak sense but in the sense of 'one exactly'. They purport it on pain of truth-value gaps; but we are well rid of that sanction. Any existence claim that is felt to inhere in the meanings of singular terms is well eliminated.

If we liked we could, alternatively, eliminate 'Socrates' as singular term by reconstruing the name as a general term true of many objects; viz., Socrates's spatiotemporal parts (cf. p. 52). For the old

[3] Thus perhaps Hochberg, "On pegasizing."

force of 'x = Socrates' can then still be recovered in paraphrase, this time as:

$(y)(y$ is a socrates if and only if y is part of $x)$.

A possible interest of this alternative is that the uniqueness of such an object x then follows from the logic of the part-whole relation, independently of any special trait of 'socrates' beyond its being true of one or more objects of the sort that can be parts.

Let us now turn our attention from names to singular descriptions. In ordinary discourse the idiom of singular description is normally used only where the intended object is believed to be singled out uniquely by the matter appended to the singular 'the' together perhaps with supplementary information to be gleaned from the context or circumstances of utterance. When we turn to canonical notation, we have to imagine that supplementary information made explicit as part of the perhaps complex sentence represented by the '$\ldots x \ldots$' of '$(\iota x)(\ldots x \ldots)$'. This eking out bears witness to what was said in § 33: that no synonymy claim is involved, and that paraphrasing depends on what we are trying to prove or find out. Eking out descriptions is pragmatic, like settling ambiguities, tenses, and indicator words. Seldom in practice do these things have to be done in full, even when we propose to reason within the outward structure of our canonical notation. We settle points that are crucial to the particular formal manoeuvres envisaged, and just imagine the rest filled in anyhow. But the logical theory which the canonical framework makes possible treats the ambiguous terms and the indicator words as having fixed references, supposed intended, even where we do not need to say which; and it treats the '$\ldots x \ldots$' of '$(\iota x)(\ldots x \ldots)$' as if it were eked out in supposedly intended ways, even when we do not need to say how. If someone is persuaded that the sentence represented by '$\ldots x \ldots$', even including all plausibly intended supplementations, is fulfilled by more than one object x or by none, then for him the question of truth or falsity of sentences containing referential occurrences of '$(\iota x)(\ldots x \ldots)$' tends, as noted at the beginning of § 37, to lapse. He will normally abstain from the discussion proper, in favor of a discussion of its propriety.

Compare now the identity '$y = (\iota x)(\ldots x \ldots)$' with the quantification:

(1) $(x)(\ldots x \ldots$ if and only if $x = y)$,

which may be briefly read '$\ldots y \ldots$ and y only'. Presumably, if either '$y = (\imath x)(\ldots x \ldots)$' or '$\ldots y \ldots$ and y only' is true of an object y, both are. Still the two formulas may diverge in their falsity conditions, on account of truth-value gaps; for these gaps may be viewed as rendering '$y = (\imath x)(\ldots x \ldots)$' innocent of truth value for each object y if not true of one, whereas '$\ldots y \ldots$ and y only' is simply false for each object y if not true of one. Hence our opposition to truth-value gaps is readily acted on: we can simply equate '$y = (\imath x)(\ldots x \ldots)$' with '$\ldots y \ldots$ and y only', thus filling the truth-value gaps of '$y = (\imath x)(\ldots x \ldots)$' with falsity. Now this move enables us to sweep singular descriptions away altogether. For, we saw earlier (§ 37) how to limit the occurrences of any singular terms other than variables to occurrences as right members of equations and as subjects of 'exists'. Where the term is '$(\imath x)(\ldots x \ldots)$', we have thereafter only to paraphrase the equations and the existence sentence away by paraphrasing '$y = (\imath x)(\ldots x \ldots)$' as '$\ldots y \ldots$ and y only', or (1), and '$(\imath x)(\ldots x \ldots)$ exists' as '$(\exists y)(\ldots y \ldots$ and y only$)$'. Such is Russell's elimination of the singular description.[4]

Singular terms other than variables have been reparsed where simple, and dissolved where of the form of descriptions. What now of that further important class of singular terms, the *algebraic* type '$\sqrt{x}$', '$x + y$', '$x + 5$', '$x + y^z$', etc.? These are the singular terms that have as their immediate constituents not sentences, as descriptions do, but further singular terms. An example not concerned with number is afforded by concatenation, § 30: '$x^\frown y$'. Another is the treatment of 'at' suggested in § 36: 'x at t'. But we can reduce this whole algebraic category to the category of descriptions, by adopting an appropriate relative term in lieu of each of the algebraic operators. For example, to get rid of '$+$' we adopt a triadic relative term 'Σ' and reckon 'Σwxy' as true when and only when $w = x + y$; thenceforward we can render anything of the form '$a + b$', however complex the terms represented by 'a' and 'b', as '$(\imath w)\Sigma wab$'. What this reduction amounts to is a reparsing of the '$=$' and '$+$' of '$w = x + y$' as a simple triadic relative term; the 'Σ' is only for vividness.

Thus '$x + y^z$' goes first into '$(\imath w)\Sigma wxy^z$'. But 'y^z' in turn goes

[4] Russell, "On denoting"; also Whitehead and Russell. By retracing the relevant reasoning of § 37 in addition to the above, the reader can see that the manner of eliminating descriptions here is really the same as Russell's despite differences in approach.

into '$(\imath u)Puyz$', where '$Puyz$' is taken as amounting to '$u = y^z$'. So '$x + y^z$' becomes '$(\imath w)\ \Sigma\ w\ x\ (\imath u)Puyz$'. Again '$x + y + z$' can be explained as '$x + (y + z)$' and hence ultimately as '$(\imath w)\ \Sigma\ w\ x\ (\imath u)\Sigma uyz$'. Similarly '$x^\frown y$' can be handled as '$(\imath w)Cwxy$', '$x^\frown y^\frown z$' as '$(\imath w)\ C\ w\ x\ (\imath u)Cuyz$', and so on.

There are still some forms of complex singular terms to consider which, like descriptions, contain sentences. Class abstraction is one that need not detain us, for we saw in (8) of § 34 how it reduces to description. As for intensional abstraction, it can be got down to description by substantially the same reparsing trick that we just now used on '$w = x + y$'. Thus, consider the brackets of propositional abstraction. Instead of viewing them as an operator that applies to a sentence to form a singular term, and then viewing '$=$' in '$a = [p]$' as a relative term that applies to the two singular terms to form a sentence, we can reparse '$= [\quad]$' as an irreducible operator that applies outright to 'a' and 'p' to form a sentence '$a = [p]$'. Thus suppose we rewrite that newly indivisible operator for vividness as 'O', so that '$a = [p]$' becomes '$a\mathrm{O}p$'; then the old '$[p]$' comes out as '$(\imath w)(w\mathrm{O}p)$'. Attribute abstraction can be treated analogously, by reparsing '$a = x[\ldots x \ldots]$' as formed by an irreducible two-place variable-binding operator in the fashion '$a\mathrm{O}_x(\ldots x \ldots)$'; then the old '$x[\ldots x \ldots]$' comes out as '$(\imath w)(w\mathrm{O}_x(\ldots x \ldots))$'. Similarly for the abstraction of relations. Thus evidently nothing stands in the way of our making a clean sweep of singular terms altogether, with the sole exception of the variables themselves.[5]

[5] Strawson, "Singular terms, ontology, and identity," pp. 446 f., 453, has supposed that demonstrative singular terms somehow defy such a program. That this is a mistake is evident from the paraphrasing of such terms into descriptions in § 34. The indicator words 'here' and 'there', with which § 34 left us, stay on as general terms; the indicator words 'now' and 'then', treated as singular terms in § 36, become reparsed as general terms. There is no evident reason to infer survival of singular terms from survival of indicator words. Cf. Russell, "Mr. Strawson on referring." Strawson's notion is doubtless causally connected somehow with an unsuccessful effort to read between my lines; he writes (p. 443), "Quine does not explicitly claim . . . elimination [of indicator words] as a merit of the recommended procedure [of eliminating singular terms]; but I think it certain that he would regard it as one." I do not. – Incidentally, let me take this opportunity to deny also the motivation hinted on p. 444, where Strawson writes, "And though I do not think he has explicitly done so, Quine might well claim [eliminations of failure of substitutivity of identity] as a further simplification to be gained by the elimination of singular terms." On the contrary, see § 35 above, especially (5), and also

That variables alone remain as singular terms may be seen as testifying to the primacy of the pronoun. One is reminded of Peirce's mot about "the noun, which may be defined as a part of speech put in place of a pronoun." [6]

Thus winnowed, what does canonical notation retain? Those of its sentences that contain no sentences as parts are composed each of a general term, without recognized internal structure (§ 36), standing in predicative position complemented by one or more variables. That is, the atomic sentences have the forms 'Fx', 'Fxy', etc. The rest of the sentences are built from the atomic ones by truth functions, quantifiers, and perhaps other devices. Three such further devices of sentence composition are the operators 'O' and 'O_x' just now noted and their analogue 'O_{xy}' for relations; but they will be reconsidered in § 44.

§ 39. DEFINITION AND THE DOUBLE LIFE

The elimination of singular terms depended on fusing '=' with some further bit of text. This does not mean that in the end we are rid of '=', along with the singular terms. For though singular terms (other than variables) are gone, '=' continues to occur flanked by variables. Like all general terms, it stays on in predicative position with variables and thus only. Indeed the very forms '. . . y . . . and y only' and '$(\exists y)(\ldots y \ldots$ and y only)', which served in § 38 to supplant the immediate contexts of descriptions, contain '$x = y$' (when spelled out according to (1) of § 38). Again the uniqueness sentences which may be expected to come into prominence with the reparsing of names require '=' flanked by variables; that one and only one thing is Socrates, e.g., runs '$(\exists y)(y$ is Socrates and y only)', or:

(1) $(\exists y)(x)(x$ is Socrates if and only if $x = y)$.

The elimination of singular terms other than variables was attended with a considerable simplification of logical theory in the closing of truth-value gaps. But now it may be feared that a comparable loss

From a Logical Point of View, pp. 144 ff., 152. These passages serve also in answer to Pap, "Belief and propositions," p. 124 n. In another paper, Strawson showed cognizance of such passages; see "A logician's landscape," pp. 234 ff., where his misgivings take other lines.

[6] Peirce, vol. 5, paragraph 153.

of simplicity is suffered on other counts. The logical laws governing '=' are applicable automatically to 'x is Socrates' *qua* 'x = Socrates', but *prima facie* irrelevant to 'x is Socrates' *qua* 'Fx'; irrelevant again to '$z = x + y$' *qua* 'Σzxy'. Moreover, inference by substitution of singular terms other than variables for variables of universal quantifications is obstructed. What would have been:

(2) If $(z)(\ldots z \ldots)$ then ... Socrates ... ,
(3) If $(z)(\ldots z \ldots)$ then $\ldots x + y \ldots$

are now:

(4) If $(z)(\ldots z \ldots)$ then $(\exists z)(z$ is Socrates and $\ldots z \ldots)$,
(5) If $(z)(\ldots z \ldots)$ then $(\exists z)(\Sigma zxy$ and $\ldots z \ldots)$.

And these, besides being cumbersome, are not even valid except on the supplementary existence premisses '$(\exists z)(z$ is Socrates)' and '$(\exists z)\Sigma zxy$'.

What seems here a complication is, however, in a way a boon. As long as singular terms other than variables are accepted at face value, the logic of quantification must somehow allow for (2) and (3); but then how exclude their 'Pegasus' analogue? What had been a tacit premiss of existence comes out into the open when, eliminating singular terms other than variables, we switch from (2) and (3) to (4) and (5).

Directness aside, no losses are sustained. It can be shown that everything that used to be demonstrable or deducible from given premisses when 'Socrates' was manipulated unquestioningly as a singular term is still demonstrable or deducible from those same premisses with the added help of the uniqueness premiss '$(\exists y)(y$ is Socrates and y only)', or (1), when 'Socrates' is reparsed as a general term. Likewise everything that could be done with '+' can still be done, in translation, in terms of 'Σ', given this uniqueness premiss for 'Σ':

(6) $(x)(y)$(if x is a number and y is a number then $(\exists z)(\Sigma zxy$ and z only)).

More generally, everything that could be done with '$(\imath x)(\ldots x \ldots)$' can still be done, in translation, on the premiss '$(\exists y)(\ldots y \ldots$ and y only)'. These supporting premisses are less to be deplored as uneconomical than prized as an unfolding of tacit assumptions, an articulation of the inarticulate.

But there is no blinking the added complications. Certainly (4) and (5) are more cumbersome than (2) and (3). Certainly '$(\exists x)(x$ is Socrates and x is Greek)', of the form '$(\exists x)(Fx$ and $Gx)$', is more cumbersome than 'Socrates is Greek' conceived as of the form 'Ga'. It is convenient to be able to bandy names as singular terms, and descriptions likewise, substituting them for variables and predicatively applying general terms to them. When we come to the shift exemplified by that from '+' to 'Σ', indeed, the loss in facility is staggering; we sacrifice precisely the moves that typify mathematics at its fleetest. Not to allow the nesting of singular terms within singular terms within singular terms without limit, in polynomial fashion, and not to allow the facile substitution of complexes for variables and equals, would diminish the power of mathematics catastrophically, even though only practically and not in principle. Happily, though, this looming dilemma can be solved.

For an endearing trait of canonical notations is that they do not bind; we can vacillate between two, opportunistically enjoying their incompatible advantages. What modern logicians call *definitions* are largely instructions for doing just that. Thus we can cleave theoretically to a canonical notation in which there are no singular terms but variables, but at the same time we can define, relative to that notation, a shorthand use of the other singular terms after all. Through the medium of such definitions we can even resuscitate (2) and (3) and the like as working rules, by showing that what they yield is, under the definitions, mere shorthand for what could be got longhand from premisses such as (1) and (6). Yet when our problems are of a kind that respond better to economy in the roots of the theory than to brevity of paraphrase and swiftness of deduction, we are still free to play the narrower canonical notation straight.

The purpose of the definitions is to enable us to lapse back into the eliminated notation, or a convenient approximation, while maintaining a key to how the strict canonical transcription would run. Substantially the appropriate definitions are therefore before us, in the very transformations already cited to show the dispensability of singular terms other than variables. Such definitions have the virtue, incidentally, of restoring singular terms in all their flexibility without reviving the nuisance of the truth-value gaps. The definition for singular description becomes simply this: write '$y = (\imath x)(\ldots x \ldots)$' and '$(\imath x)(\ldots x \ldots)$ exists' as notational variants of '$\ldots y \ldots$ and y only' and '$(\exists y)(\ldots y \ldots$ and y only)', and, harking back now to the reasoning in § 37, write '---$(\imath x)(\ldots x \ldots)$---' as an abbreviation of:

(7) $(\exists y)(y = (\imath x)(\ldots x \ldots)$ and ---y---).

(In this exposition we understand '---y---' as an arbitrary open sentence, and '---$(\imath x)(\ldots x \ldots)$---' as the same with the singular description in place of 'y'.)

The three parts of the above definition suffice, when applied successively and in iteration, to restore '$(\imath x)(\ldots x \ldots)$' to every position of occurrence of which a free variable is capable. Actually the definition needs some tightening, in known ways.[1] But enough; logic students know this logic of descriptions, essentially Russell's, and my purpose has been rather a philosophical clarification of its business.

Along with the practical resuscitation thus of descriptions, the various other singular terms that were reduced to descriptions in § 38 are resuscitated too—notably the terms of algebraic type. This is a convenient channel also for reintroducing names as singular terms: 'Socrates' as singular term can be defined as '$(\imath x)$(x is Socrates)' on the basis of 'Socrates' as general term. In practice the defined singular term 'Socrates' and the defining general term 'Socrates' would doubtless be rendered distinctively, say by lowering one initial 'S' to 's' or even by leaving the singular term as '$(\imath x)$(x is Socrates)'. Technically we could keep 'Socrates' for both, since in a well-formed notation the positions open to general and singular terms will be mutually exclusive and so avert any ultimate ambiguity. But what is particularly to be noted is that it would be pointless to defend either the singular term or the general term as *the* regular counterpart of the name 'Socrates' of ordinary language. In paraphrasing some sentences for some purposes the singular term will come in handy; on other occasions the general term works better. We are reminded once again that paraphrase makes no synonymy claim. As for the epithet 'name', it applies first and foremost to the 'Socrates' of ordinary language and derivatively to either of its formalizations; when our intention is more specific we can say so, as e.g. in the provisional convention at the end of § 37.

The virtues of definition as a method of eating one's cake and having it are strikingly illustrated by quotation and concatenation. Quotation, which produces names of linguistic forms by picture-writing, has the overwhelming practical convenience of visible reference. But it has the drawback, for some purposes of systematic

[1] Thus see § 37, note 1.

theory, that the names which it produces are, regardless of length, logically unstructured. Hence the main virtue of spelling, which comes properly apart. Spelling, however protracted, analyzes into iteration of the one little two-term algebraic operation of concatenation plus a small stock of names of letters. Spelling also has the virtue of shattering the non-referential occurrences of terms which quotation engenders (cf. § 30); but this is an incidental surface effect. Then, finally, there is the elimination in turn of concatenation in favor of the triadic relative term 'C'. The advantage here is theoretical simplicity, through the elimination of complex singular terms; and the disadvantage is cumbersomeness, the sacrifice of algebraic facility. Now thanks to the device of definition we can enjoy each of these benefits without foreswearing the others. We theorize within the 'C' theory secure in the knowledge that the luxuries of spelling and even of quotation can be restored, by definition, at will. It is one of the consolations of philosophy that the benefit of showing how to dispense with a concept does not hinge on dispensing with it.