FORMULA SHEET
Experimental Design Comprehensive Exam

TESTS TO COMPARE PAIRS OF MEANS

Tukey's: $T_\alpha = q_\alpha(a, df_E)\sqrt{MS_E/n}$

Fisher's (LSD): $LSD = t_{\alpha/2, df_E}\sqrt{2MS_E/n}$

Duncan's: $R_p = r_\alpha(p,\, df_E)\sqrt{MS_E/n}$

TEST FOR CONTRASTS

Contrast: $t = \frac{\sum_i c_i y_{i.}}{\sqrt{nMS_E \sum_i c_i^2}}$

Orthogonal Contrasts:

$SS_C = \frac{(\sum_i c_i y_{i.})^2}{n \sum_i c_i^2}$

THE BALANCED INCOMPLETE BLOCK DESIGN

$SS_{tr(adj)} = \frac{k \sum_i Q_i^2}{\lambda a}$ where $Q_i = y_{i.} - \frac{1}{k}\sum_j n_{ij} y_{.j}$

$SS_{bl} = \sum_j \frac{y_{.j}^2}{k} - \frac{y_{..}^2}{N}$

$SS_{tr} = \sum_i \frac{y_{i.}^2}{r} - \frac{y_{..}^2}{N}$

$SS_{bl(adj)} = SS_{tr(adj)} + SS_{bl} - SS_{tr}$

THE 2^k DESIGN

$A = \frac{\text{Contrast}_A}{n\, 2^{k-1}}$

$SS_A = \frac{(\text{Contrast}_A)^2}{n\, 2^k}$

THE TWO-FACTOR FACTORIAL DESIGN

$$SS_A = \sum_i \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}, \quad SS_B = \sum_j \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = \sum_i \sum_j \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn} - SS_A - SS_B$$

$\mathbb{E}(MS)$	A Fixed B Fixed	A Fixed B Random	A Random B Random
$\mathbb{E}(MS_A)$	$\sigma^2 + \frac{bn\sum \tau_i^2}{a-1}$	$\sigma^2 + n\sigma_{\tau\beta}^2 + \frac{bn\sum_{i=1}^a \tau_i^2}{a-1}$	$\sigma^2 + n\sigma_{\tau\beta}^2 + bn\sigma_\tau^2$
$\mathbb{E}(MS_B)$	$\sigma^2 + \frac{an\sum \beta_j^2}{b-1}$	$\sigma^2 + an\sigma_\beta^2$	$\sigma^2 + n\sigma_{\tau\beta}^2 + an\sigma_\beta^2$
$\mathbb{E}(MS_{AB})$	$\sigma^2 + \frac{n\sum\sum(\tau\beta)_{ij}^2}{(a-1)(b-1)}$	$\sigma^2 + n\sigma_{\tau\beta}^2$	$\sigma^2 + n\sigma_{\tau\beta}^2$

THE TWO-STAGE NESTED DESIGN

$$SS_A = \frac{1}{bn}\sum_{i=1}^a y_{i..}^2 - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \frac{1}{n}\sum_{i=1}^a \sum_{j=1}^b y_{ij.}^2 - \frac{1}{bn}\sum_{i=1}^a y_{i..}^2 \text{ with } df = a(b-1)$$

$\mathbb{E}(MS)$	A Fixed B Fixed	A Fixed B Random	A Random B Random
$\mathbb{E}(MS_A)$	$\sigma^2 + \frac{bn\sum \tau_i^2}{a-1}$	$\sigma^2 + n\sigma_\beta^2 + \frac{bn\sum \tau_i^2}{a-1}$	$\sigma^2 + n\sigma_\beta^2 + bn\sigma_\tau^2$
$\mathbb{E}(MS_{B(A)})$	$\sigma^2 + \frac{n\sum\sum \beta_{j(i)}^2}{a(b-1)}$	$\sigma^2 + n\sigma_\beta^2$	$\sigma^2 + n\sigma_\beta^2$

THE SPLIT-PLOT DESIGN

$$\mathbb{E}(MS_{blocks}) = \sigma^2 + ab\sigma_\tau^2, \quad \mathbb{E}(MS_A) = \sigma^2 + b\sigma_{\tau\beta}^2 + \frac{rb\sum \beta_j^2}{a-1},$$

$$\mathbb{E}(MS_{blocks\times A}) = \sigma^2 + b\sigma_{\tau\beta}^2, \quad \mathbb{E}(MS_B) = \sigma^2 + \frac{ra\sum \gamma_k^2}{ab-1},$$

$$\mathbb{E}(MS_{AB}) = \sigma^2 + \frac{r\sum\sum(\beta\gamma)_{jk}^2}{(a-1)(b-1)}$$