with (δ, γ) a point on $L(6,2; x, y)$. For (x, y) close to $(6,2)$,

$$\frac{x}{y} \doteq 3 + \frac{1}{2}x - \frac{3}{2}y$$

The graph of $z = 3 + \frac{1}{2}x - \frac{3}{2}y$ is the plane tangent to the graph of $z = x/y$ at $(x, y, z) = (6,2,3)$.

Some mathematical notation There are several concepts that are taken up in this text that are needed in a simpler form in the earlier chapters. These include results on divided differences of functions, vector spaces, and vector and matrix norms. The minimum necessary notation is introduced at this point, and a more complete development is left to other more natural places in the text.

For a given function $f(x)$, define

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \qquad f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \tag{1.1.13}$$

assuming x_0, x_1, x_2 are distinct. These are called the first- and second-order divided differences of $f(x)$, respectively. They are related to derivatives of $f(x)$:

$$f[x_0, x_1] = f'(\xi) \qquad f[x_0, x_1, x_2] = \tfrac{1}{2}f''(\zeta) \tag{1.1.14}$$

with ξ between x_0 and x_1, and ζ between the minimum and maximum of x_0, x_1, and x_2. The divided differences are independent of the order of their arguments, contrary to what might be expected from (1.1.13). More precisely,

$$f[x_0, x_1] = f[x_1, x_0]$$

$$f[x_0, x_1, x_2] = f[x_i, x_j, x_k] \tag{1.1.15}$$

for any permutation (i, j, k) of $(0,1,2)$. The proofs of these and other properties are left as problems. A complete development of divided differences is given in Section 3.2 of Chapter 3.

The subjects of vector spaces, matrices, and vector and matrix norms are covered in Chapter 7, immediately preceding the chapters on numerical linear algebra. We introduce some of this material here, while leaving the proofs till Chapter 7. Two vector spaces are used in a great many applications. They are

$$\mathbf{R}^n = \left\{ x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \middle|\, x_1, \ldots, x_n \text{ real numbers} \right\}$$

$$C[a, b] = \{ f(t) \mid f(t) \text{ continuous and real valued}, a \le t \le b \}$$