8. Consider the second-order divided difference $f[x_0, x_1, x_2]$ defined in (1.1.13).

(a) Prove the property (1.1.15), that the order of the arguments x_0, x_1, x_2 does not affect the value of the divided difference.

(b) Prove formula (1.1.14),

$$f[x_0, x_1, x_2] = \tfrac{1}{2} f''(\zeta)$$

for some ζ between the minimum and maximum of x_0, x_1, and x_2. *Hint:* From part (a), there is no loss of generality in assuming $x_0 < x_1 < x_2$. Use Taylor's theorem to reduce $f[x_0, x_1, x_2]$, expanding about x_1; and then use the intermediate value theorem to simplify the error term.

(c) Assuming $f(x)$ is twice continuously differentiable, show that $f[x_0, x_1, x_2]$ can be extended continuously to the case where some or all of the points x_0, x_1, and x_2 are coincident. For example, show

$$f[x_0, x_1, x_0] \equiv \operatorname*{Limit}_{x_2 \to x_0} f[x_0, x_1, x_2]$$

exists and compute a formula for it.

9. (a) Show that the vector norms (1.1.16) and (1.1.18) satisfy the three general properties of norms that are listed following (1.1.18).

(b) Show $\|x\|_2$ in (1.1.17) is a vector norm, restricting yourself to the $n = 2$ case.

(c) Show that the matrix norm (1.1.19) satisfies (1.1.20) and (1.1.21). For simplicity, consider only matrices of order 2×2.

10. Convert the following numbers to their decimal equivalents.

(a) $(10101.101)_2$ (b) $(2A3.FF)_{16}$ (c) $(.101010101\ldots)_2$

(d) $(.AAAA\ldots)_{16}$ (e) $(.00011001100110011\ldots)_2$

(f) $(11\ldots1)_2$ with the parentheses enclosing n 1s.

11. To convert a positive decimal integer x to its binary equivalent,

$$x = (a_n a_{n-1} \ldots a_1 a_0)_2$$