CECS 419-519, Writing Assignment 8, Due 8:00 am, April 19th, 2024, Dr. Ebert

Directions

Make sure name is on all pages. Order pages (front and back) so that solutions are presented in their original numerical order. **Please no staples or folding of corners (your papers won't get lost). A paper clip is OK** Show all necessary work and substantiate all claims. Avoid plagiarism.

Problems

1. Prove that there is some constant c such that, for every strings x and y,

$$K(xy) \leq 2K(x) + K(y) + c,$$

   where xy denotes the concatenation of x with y. Hint: self-terminating code. (20 pts)

2. With the help of the Turing-machine model of computation, we may provide formal definitions for the complexity classes `P` and `NP`. For example language $L \in$ `P` iff there exists a DTM M and a polynomial $p(n)$ for which i) $L(M) = L$ and ii) for an input x of size $n \geq 0$, the computation $M(x)$ requires at most $p(n)$ steps. Similarly, $L \in$ `NP` iff there exists an NTM N and a polynomial $p(n)$ for which i) $L(N) = L$ and ii) for an input x of size $n \geq 0$, each branch of the computation tree $T(N, x)$ has a length not exceeding $p(n)$. Show that any language L that satisfies the latter definition is a member of `NP` in accordance with the definition for `NP` provided in the Computational Complexity lecture.

   (a) For a given instance x of L describe a certificate in relation to x. Hint: you may assume that, for the δ transition function of the NTM N that decides L, $|\delta(q, s)| \leq 2$, for all $q \in Q$ and $s \in \Gamma$. (10 pts)

   (b) Provide a semi-formal verifier algorithm that takes as input i) an instance x of L, ii) a certificate for x as defined in part a, and decides if the certificate is valid for x. (10 pts)

   (c) Provide the running time (as a function of size parameter $n = |x|$) of your verifier algorithm. Defend your answer. (5 pts)

3. For each decision problem L defined below, provide a complexity class C for which the statement $L \in C$ is true. Moreover, C should be chosen so that it is the best possible, meaning that it is the least complex of all possible valid choices. For example, if L is in both `P` and `NP`, then `P` is the better choice. The choices for C are `P`, `NP`, `co-NP`, Σ_2^p, and Π_2^p. Provide a predicate-logic-formula characterization for each (define all certificate sets and predicate functions). (5 pts each)

   (a) An instance of `Sum-Free Hamilton Path` consists of a simple graph $G = (V, E)$ and a nonnegative integer t. Moreover, each of the n vertices of G is labeled with a positive integer. The problem is to decide if i) G has a Hamilton path $P = v_1, v_2, \ldots, v_{n-1}, v_n$ for which the set of labels $S = \{\text{label}(v_1), \ldots, \text{label}(v_{n/2})\}$ has no subset that sums to t.

(b) An instance of `Satisfied Max Cut` is a simple graph $G = (V, E)$ and a nonnegative integer $k \geq 0$, where each vertex v of G is labeled with a Boolean formula F_v. The problem is to decide if, for every cut of size k (See the `Max Cut` problem from Writing Assignment 6), there is at least one blue vertex and one red vertex each of which is labeled with a satisfiable formula.

(c) An instance of `Unsolvable Quadratic Diophantine` consists of three positive integers $a, b, c > 0$ and the problem is to decide if the equation $ax^2 + by = c$ has no positive integer solutions, meaning there is no pair (x, y) of positive integers that satisfies the equation.

(d) Recall the Tseytin transformation f : `SAT` $\rightarrow$ `3SAT` used to map reduce `SAT` to `3SAT`. An instance of decision problem L is a Boolean formula F and a nonnegative integer k. The problem is to decide if $f(F)$ has at least k clauses.

4. For the CFG provided in Example 2.3 of the Space Complexity lecture, provide a derivation of the expression

$$((a \times a)) + (a + a \times a).$$

(10 pts)

5. Provide a CFG G for which $L(G)$ equals all words of the form $u\#v$, where $u, v \in \{a, b\}^*$ and $u \neq v$. Defend your solution by explaining why u can never equal v, and that all the words that should be in $L(G)$ can in fact be generated. (20 pts)