

CECS 419-519, Writing Assignment 4, Due 8:00 am, February 23rd,  
2024, Dr. Ebert

## Directions

Make sure name is on all pages. Order pages (front and back) so that solutions are presented in their original numerical order. **Please no staples or folding of corners (your papers won't get lost). A paper clip is OK** Show all necessary work and substantiate all claims. Avoid plagiarism.

## Problems

1. Prove that there is a constant  $c$  such that, for all  $x, y \in \Sigma^*$ ,  $0 \leq K(x|y) \leq K(x) + c$ .

2. Prove that

$$\sum_{|x|=n} 2^{-2K(x|n)}$$

is bounded by some positive constant  $C$  that is independent of  $n$ . Hint: use the ideas from Proposition 3.1.

3. A binary word  $x$  is said to be **Kolmogorov random** iff  $K(x|n) \geq n$ , where  $n = |x|$ . Prove that, if  $n$  is sufficiently large and  $x$  has  $n/4$  1's and  $3n/4$  0's, then  $x$  cannot be Kolmogorov random. Hint: consider Huffman coding!

4. Provide the recursive self-terminating encoding for the number 337.

5. Use the **self** programming concept to prove that  $K(x)$  is not a URM computable function. Hint: review Theorem 3 and the definition of *recursively enumerable* on pages 17-19.