

# CECS 419-519, Writing Assignment 1, Due 8:00 am, February 2nd, 2024, Dr. Ebert

## Directions

Make sure name is on all pages. Order pages (front and back) so that solutions are presented in their original numerical order. **Please no staples or folding of corners (your papers won't get lost)**. Show all necessary work and substantiate all claims. Avoid plagiarism.

## Problems

1. Do the following.
  - (a) Provide the instructions of the URM program  $P$  that computes the function  $f(n) = n^2$ . Use the online URM simulator to verify that your URM is correct. (10 pts)
  - (b) Briefly describe the idea behind your program and the purpose of each register. (10 pts)
2. Prove that the  $\pi$  encoding function described in lecture is a one-to-one correspondence between  $\mathcal{N} \times \mathcal{N}$  and  $\mathcal{N}$ . Hint: for each  $z \in \mathcal{N}$ , show that there is a unique pair  $(x, y) \in \mathcal{N} \times \mathcal{N}$  for which  $\pi(x, y) = z$ . (15 pts)
3. Do the following.
  - (a) Provide the Gödel number  $e$  for the program  $P = J(1, 2, 7), S(2), S(2), S(2), S(3), J(1, 1, 1), T(3, 1)$ . Write  $e$  as a sum of powers of two minus one. (10 pts)
  - (b) Apply the decoding functions to  $e$  in order to verify that your answer to part a is correct. Show all work. (10 pts)
  - (c) Describe  $\phi_e(x)$ ,  $W_e$ , and  $E_e$ . Justify your answers. (10 pts)
4. Prove that there is a total computable function  $h(n)$  for which, for all  $n \geq 0$ ,  $\phi_{h(n)}(x) = x^n$ . Hint: you may assume that  $x^n$  is a computable function of two variables. (15 pts)