CECS 419-519, Writing Assignment 1, Due 8:00 am, February 2nd, 2024, Dr. Ebert

Directions

Make sure name is on all pages. Order pages (front and back) so that solutions are presented in their original numerical order. **Please no staples or folding of corners (your papers won't get lost).** Show all necessary work and substantiate all claims. Avoid plagiarism.

Problems

1. Do the following.

   (a) Provide the instructions of the URM program P that computes the function $f(n) = n^2$. Use the online URM simulator to verify that your URM is correct. (10 pts)

1. J(1,2,9)
2. Z(3)
3. J(1,3,7)
4. S(3)
5. S(4)
6. J(1,1,3)
7. S(2)
8. J(1,1,1)
9. J(4,1)

(b) Briefly describe the idea behind your program and the purpose of each register. (10 pts)

Add X X times to R_4

R_2: counts the number of times x has been added to R_4

R_3: Counts up to X

R4: Counts up to X^2

2. Prove that the π encoding function described in lecture is a one-to-one correspondence between $\mathcal{N} \times \mathcal{N}$ and $\mathcal{N}$. Hint: for each $z \in \mathcal{N}$, show that there is a unique pair $(x, y) \in \mathcal{N} \times \mathcal{N}$ for which $\pi(x, y) = z$. (15 pts)

By the Fundamental Theorem of Arithmetic every positive Integer z has a unique representation of the form $z = 2^x p_1^{\alpha_1} \cdots p_k^{\alpha_k}$, where $p_1, \ldots, p_k$ are odd primes and $x \geq 0$! Moreover, since the product of two odd numbers is odd, it follows that $p_1^{\alpha_1} \cdots p_k^{\alpha_k}$ is an odd number of the form $2y+1$, for some $y \in \mathbb{N}$. Thus $\pi'(x,y) = 2^x(2y+1)$ is a one-to-one corr. between $\mathbb{N} \times \mathbb{N}$ and $\mathbb{N}^+$. Now let $\pi(x,y) = \pi'(x,y) - 1$

3. Do the following.

(a) Provide the Gödel number e for the program $P = J(1, 2, 7), S(2), S(2), S(2), S(3), J(1, 1, 1), T(3, 1)$. Write e as a sum of powers of two minus one. (10 pts)

$\beta(J(1,2,7)) = 4\xi(0,1,6) + 3 = 4\pi(2,6) + 3 = (4)(51) + 3 = 207$. Also, $\beta(S(2)) = (4)(1) + 1 = 5$

$\beta(S(3)) = (4)(2) + 1 = 9$, $\beta(J(1,1,1)) = 4(\xi(0,0,0)) + 3 = 3$, and $T(3,1) = 4\pi(2,0) + 2 = 14$.

$\therefore e = 2^{207} + 2^{213} + 2^{219} + 2^{225} + 2^{235} + 2^{239} + 2^{254} - 1$

(b) Apply the decoding functions to e in order to verify that your answer to part a is correct. Show all work. (10 pts)

$\tau^{-1}(e) = \beta^{-1}(207), \beta^{-1}(5), \beta^{-1}(5), \beta^{-1}(5), \beta^{-1}(9), \beta^{-1}(3), \beta^{-1}(14) = J(1,2,7), S(2), S(2), S(2), S(3), J(1,1,1), T(3,1).$ ✓

For example, $(207-3)/4 = 51$. $51+1 = 52 = 2^2((2)(6)+1)$

Also, $2+1 = 3 = 2^0((2)(1)+1) \Rightarrow$
$J(0+1, 1+1, 6+1) = J(1,2,7)$

(c) Describe $\phi_e(x)$, W_e, and E_e. Justify your answers. (10 pts)

$$\phi_e(x) = \begin{cases} \frac{x}{3} & \text{if } 3 \mid x \\ \uparrow & \text{otherwise} \end{cases}$$

True since $R_1 = R_2$ iff R_1 is a multiple of 3. Thus, $J(1,2,7)$ Jumps out of program iff $3 \mid x$. Otherwise program loops forever. $W_e = \{0, 3, 6, \ldots\}$
$E_e = \{0, 1, 2, \ldots\}$

4. Prove that there is a total computable function $h(n)$ for which, for all $n \geq 0$, $\phi_{h(n)}(x) = x^n$. Hint: you may assume that x^n is a computable function of two variables. (15 pts)

By the S-M-N Theorem applied to the computable function $g(n, x) = x^n$, there is a total computable function $h(n)$ for which

$\phi_{h(n)}(x) = x^n$, for all $n \geq 0$.

Thus $\phi_{h(0)}, \phi_{h(1)}, \ldots$ lists all the power functions.