Directions: Turn in handwritten solutions on the morning of the May 16th final exam. Order your solutions by problem number/letter, and do *not* write on the back of any page. Only the front side of each page will be read. Please do *not* staple or fold your pages. Use a paper clip if necessary. Although it's OK to discuss problems with other students, plagiarism will not be tolerated and will result in a final course grade of F. Make sure to describe solutions in your own words. All problems are equally weighted and must be passed in order to pass LO12.

Problems

1. Use the union, concatenation, and star closure properties to construct an NFA that accepts the language L(E), where

 $E = (01 \cup 10)(111)^*(01 \cup 10).$

See Example 26 of the Finite Automata lecture. (5 pts)

- 2. Provide a DFA that accepts the language L of all binary words that have a number of 1's that is divisible by 3 (yes, 0 is divisible by 3). Then apply the algorithm described in Section 7 of the Finite Automata lecture to determine a regular expression E for which L = L(E). See Example 27 of the Finite Automata lecture. (10 pts)
- 3. Do the following.
 - (a) Provide a context free grammar $G = (V, \Sigma, R, S)$ for which L(G) is the set of binary words that have an odd number of 0's and exactly one 1. Please clearly define V, Σ , and R for your CFG. (10 pts)
 - (b) Use G to provide a leftmost derivation of w = 000100. (5 pts)
- 4. Do the following.
 - (a) Provide the state diagram of a Turing machine that accept all words of the form $w_1 \# w_2$, where w_1 and w_2 are binary strings, each having the same number of 1's. Hint: words such as #, 00#, and #000 should be accepted. Use the tape alphabet $\Gamma = \{0, 1, \#, x\}$. Test your program at

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https://turingmachinesimulator.com/
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(10 pts)

(b) Show the computations for inputs a) 10#01 and b) 10#11. Verify that the former is an accepting computation, while the latter is a rejecting computation. (5 pts)