CECS 329, LO9 Assessment, April 18th, Spring 2024, Dr. Ebert

Problems

LO5. Solve the following.

(a) Provide the instructions of a URM program that computes the predicate function $f(x, y) = (x > y)$. For example, $f(5, 3) = 1$, while $f(3, 3) = f(4, 7) = 0$.

(b) Describe the role each register plays in computing $f(x, y)$.

LO6. Solve the following problems.

(a) Provide the URM program P whose Gödel number equals

$$2^{26} + 2^{48} + 2^{96} + 2^{113} - 1.$$

Show all work.

(b) A universal program P_U is simulating a program that has 286 instructions and whose Gödel number is

$$x = 2^3 + 2^{179} + 2^{192} + 2^{215} + 2^{223} + 2^{275} + \cdots + 2^{c_{286}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^2 + 2^6 + 2^8 + 2^{15} + 2^{19} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO7. Answer the following.

(a) Describe what it means to be a positive instance of the `Self Accept` decision problem.

(b) In applying the diagonalization method towards proving the undecidability of `Self Accept`, we defined a computable function $g(x)$ in terms of $f(x)$, where $f(x)$ is the decision function for `Self Accept`, which we assume to be total computable. Provide the formula for computing $g(x)$ and describe what needs to be established about $g(x)$ in order for the diagonalization method to be successful.

(c) Suppose URM program P_{1269} computes $\phi_{1269}(x)$ and that

$$\phi_{1269}(x) = x \bmod 2.$$

What is the value of $g(1269)$ and how does this prove that $g \neq \phi(1269)$? Defend your answer

LO8. An instance of the decision problem `Thousand-Plus Range` is a Gödel number x, and the problem is to decide if function ϕ_x has a non-empty range and whose range members have a value of at least 1000. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } \phi_x \text{ has a thousand-plus range} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where e_1 is the Gödel number of the program that computes the function $\phi_{e_1}(y) = 4000y^2 + 300y$.

ii. $x = e_2$, where e_2 is the Gödel number of the program that computes the function $\phi_{e_2}(y) = 1459$.

iii. $x = e_3$, where e_3 is the Gödel number of the program that computes the function $\phi_{e_3}(y) = 2^{2^{4(y+1)}}$.

(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not ϕ_x has a thousand-plus range. Do this by writing a program P that uses g and makes use of the `self` programming construct. Then use a proof by cases to show that P creates a contradiction.

LO9. Solve the following.

(a) Provide the state diagram for a DFA M that accepts all binary words having an even number of 0's *and* does *not* contain the subword 101. Hint: make sure that, for each state of M, you are able to describe the current status of *both* properties in relation to being in that state.

(b) Show the computation of M on inputs i) $w_1 = 1100100$ and ii) $w_2 = 0110100$.