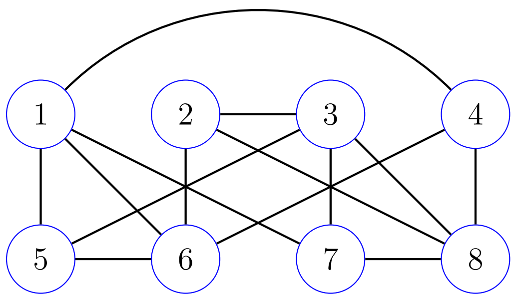
CECS 329, LO Assessment 7, March 21st, Spring 2024, Dr. Ebert

Problems

LO3. Solve the following.

(a) Suppose the graph G shown below is an instance of the `Hamilton Cycle` decision problem. Draw $f(G)$, where f is the mapping reduction from `HC` to the `Traveling Salesperson` decision problem. Hint: remember the k value!



(b) Verify that the mapping is valid by verifying that G and $f(G)$ are either both positive or both negative instances. Defend your answer.

LO4. Answer the following. Note: correctly answering two of the three constitutes a pass.

(a) Write either P, NP, or co-NP next to each of the following decision problems in terms of which one best characterizes the complexity of the problem. Note: there is only one best answer for each. Two points each.

i. An instance of the `Vertex Cover` decision problem is a pair (G, k), where $G = (V, E)$ is a simple graph, $k \geq 0$ is an integer, and the problem is to decide if G has a **vertex cover** of size k, i.e. a set $C \subseteq V$ for which every edge $e \in E$ is incident with at least one vertex in C.

ii. An instance of `Substring` is a pair (s_1, s_2) of binary strings and the problem is to decide if s_1 occurs as a substring of s_2. For example $(10101, 001010111)$ is a positive instance of `Substring` since 10101 is a substring of 001010111.

iii. An instance of `Bounded Independent Set` is a graph $G = (V, E)$ and an integer $k \geq 0$ and the problem is to decide if the maximum independent set in G is of a size that does not exceed k.

(b) Describe the three main steps that must be completed in order to establish that a decision problem L is a member of `NP`. Clearly define all technical terms. Hint: your three steps should make reference to two different technical terms that need defining.

(c) Provide the chain of polynomial-time mapping reductions provided in lecture that establishes the `NP`-completeness of `TSP`. Hint:

`Undirected Hamilton Path (HP)` $\leq_m^p$ `Hamilton Cycle (HC)`

is one of the reductions.

LO5. Solve the following.

(a) Provide the instructions of a URM program that computes the function $f(x) = 2^x$.

(b) Describe the role each register plays in computing $f(x)$.

LO6. Solve the following problems.

(a) Provide the URM program P whose Gödel number equals

$$2^{55} + 2^{73} + 2^{114} + 2^{133} - 1.$$

Show all work.

(b) A universal program P_U is simulating a program that has 286 instructions and whose Gödel number is

$$x = 2^3 + 2^{179} + 2^{191} + 2^{215} + 2^{223} + 2^{275} + \cdots + 2^{c_{286}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^3 + 2^7 + 2^{13} + 2^{16} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO7. Answer the following. Note: correctly answering two of the three constitutes a pass.

(a) Describe what it means to be a positive instance of the `Total` decision problem.

(b) In applying the diagonalization method towards proving the undecidability of `Total`, we defined a computable function $g(x)$ in terms of $f(x)$, the decision function for `Total`, which we assume to be total computable. Provide the formula for computing $g(x)$ and describe what needs to be established about $g(x)$ in order for the diagonalization method to be successful.

(c) Suppose URM program P_{51} computes $\phi_{51}(x)$ and that

$$\phi_{51}(x) = \begin{cases} \lfloor x/2 \rfloor & \text{if } x \text{ is even} \\ \uparrow & \text{otherwise} \end{cases}$$

What is the value of $g(51)$? Defend your answer.