CECS 329, Learning Outcome Assessment 6, March 14th, Spring 2024, Dr. Ebert

Problems

LO2. An instance of the `Max Cut` decision problem is a simple graph $G = (V, E)$ and an integer $k \geq 0$. The problem is to decide if there is a way to color the vertices of G using the colors red and blue and results in there being at least k edges $e = (u, v)$ for which u and v are assigned different colors.

(a) For a given instance (G, k) of `Max Cut`, describe a certificate in relation to (G, k).

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (G, k) of `Max Cut`, ii) a certificate for (G, k) as defined in part a, and decides if the certificate is valid for (G, k).

(c) Provide size parameters that may be used to measure the size of an instance (G, k) of `Max Cut`.

(d) Use the size parameters from part c to describe the running time of your verifier from part b. Defend your answer in relation to the algorithm you provided for the verifier.

LO3. An instance $\mathcal{C}$ of `3SAT` consists of clauses $c_1 = (x_1, \overline{x}_2, x_3)$, $c_2 = (\overline{x}_2, x_3, x_4)$, $c_3 = (\overline{x}_1, x_2, \overline{x}_4)$, and $c_4 = (\overline{x}_1, \overline{x}_3, x_4)$. Answer the following questions about the mapping reduction $f(\mathcal{C}) = (G, k)$ provided in lecture from `3SAT` to `Clique` and applied to instance $\mathcal{C}$.

(a) How many vertices and edges does G have? Explain and show work. Hint: there are six different vertex-group pairs.

(b) What is the value of k?

(c) Given that $\alpha = (x_1 = x_2 = 0, x_3 = 1, x_4 = 0)$ satisfies $\mathcal{C}$, provide a clique set for G that certifies (G, k) is a positive instance of `Clique`. Hint: for each clique member, indicate the group from which it came.

LO4. Answer the following questions. Correctly answering at least two of the three is sufficient for passing

(a) Provide the definition of what it means for a decision problem to be `NP`-complete.

(b) Describe the three main steps that must be completed in order to establish that a decision problem L is a member of `NP`. Clearly define all technical terms.

(c) Each of the following graph decision problems described below takes as input a simple graph $G = (V, E)$ and a nonnegative integer $k \geq 0$. Classify each one as either being in `P`, `NP`, or co-`NP`.

i. Decide if G has fewer than k connected components.

ii. Decide if the size of every independent set of G is less than or equal to k.

iii. Decide if G has a vertex cover of size k.

LO5. Provide the instructions of a URM program that computes the function $f(x) = x^2$. For each register used, provide a few sentences describing how it is used.

LO6. Solve the following problems.

(a) Provide the URM program P whose Gödel number equals

$$2^{22} + 2^{35} + 2^{65} + 2^{133} - 1.$$

Show all work.

(b) A universal program P_U is simulating the computation of a program P_x on some input y, where P_x that has 136 instructions and whose Gödel number is

$$x = 2^7 + 2^{31} + 2^{43} + 2^{55} + 2^{65} + 2^{109} + \cdots + 2^{c_{136}} - 1.$$

If the current configuration of the computation has encoding

$$\sigma = 2^5 + 2^{10} + 2^{18} + 2^{24} - 1,$$

then provide the next configuration of the computation *and* its encoding.