CECS 329, LO11 Assessment, May 2nd 2024, Dr. Ebert

Problems

LO7. Answer the following.

(a) Describe what it means for natural number x to be a positive instance of the `Total` decision problem.

Solution. See lecture notes.

(b) In applying the diagonalization method towards proving the undecidability of `Total`, we defined a computable function $g(x)$ in terms of $f(x)$, where $f(x)$ is the decision function for `Total`, which we assume to be total computable. Provide the formula for computing $g(x)$ and describe what needs to be established about $g(x)$ in order for the diagonalization method to be successful.

Solution. See lecture notes.

(c) Suppose URM program P_{643} computes $\phi_{643}(x)$ and that

$$\phi_{643}(x) = \begin{cases} 0 & \text{if } x \leq 700 \\ \uparrow & \text{otherwise} \end{cases}$$

Explain why we are sure that $g \neq \phi_{643}$.

Solution. Since $g(x)$ is total and $\phi_{643}(x)$ is not total, it must be the case that $g \neq \phi_{643}$.

LO8. An instance of the decision problem `Infinite Range` is a Gödel number x, and the problem is to decide if the function $\phi_x(y)$ has an infinite range. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } \phi_x(y) \text{ has an infinite range} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where $\phi_{e_1}(y) = y^2 \bmod 101$.
Solution.

$E_{e_1} = \{0, \ldots, 100\}$

$g(e_1) = 0$ which is a finite set

ii. $x = e_2$, where e_2 is the Gödel number of the program $P_{e_2} = S(1), S(1), J(1, 2, 1)$.
Solution.

$\phi_{e_2}(y) = y + 2$

$E_{e_2} = \mathbb{N}$

so $g(e_2) = 1$

iii. $x = e_3$, where

$$\phi_{e_3}(y) = \begin{cases} y^3 & \text{if } y \leq 500 \\ \uparrow & \text{otherwise} \end{cases}$$

Solution.

$g(e_3) = 0$

since $E_{e_3} = \{0, 1, 8, 27, \ldots, 500^3\}$ is finite.

(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not $\phi_x(y)$ has an infinite range. Do this by writing a program P that uses g and makes use of the `self` programming construct. Then use a proof by cases to show that P creates a contradiction.

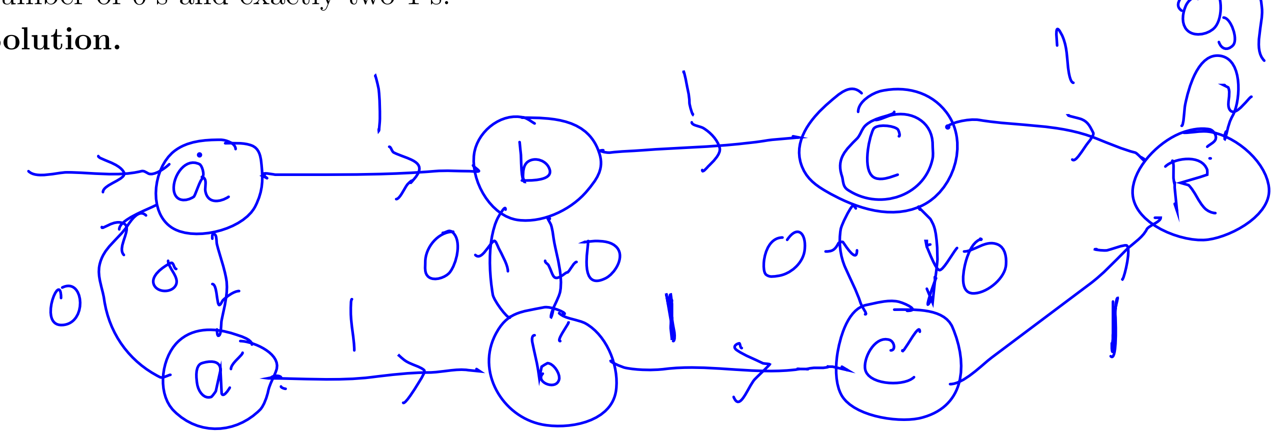
Solution.

```
P
Input y
If (g(self) = 1)
    Return 0
Else // g(self) = 0
    Return y
```

LO9. Solve the following.

(a) Provide the state diagram for a DFA M that accepts a binary word w iff w has an even number of 0's and exactly two 1's.

Solution.



(b) Show the computation of M on inputs i) $w_1 = 001001$ and ii) $w_2 = 010101$.

Solution.

0 0 1 0 0 1

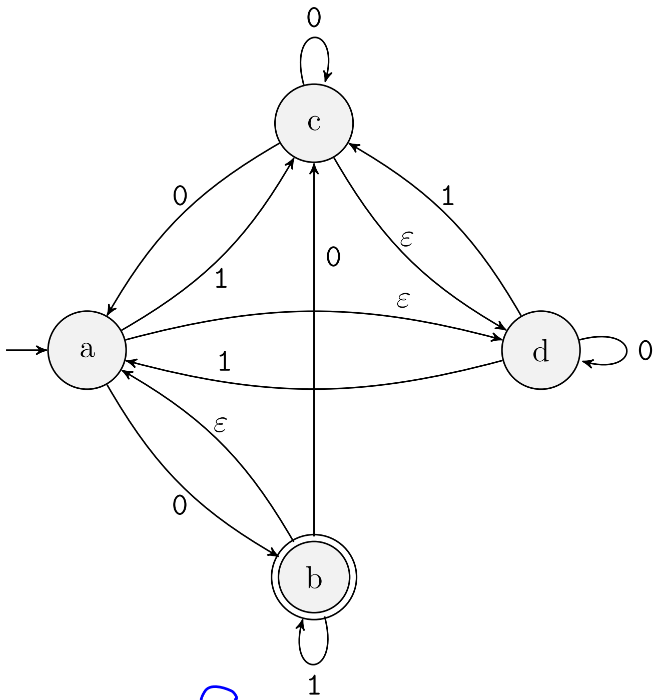
a a' a b b' b $\boxed{C}$ ← accept

0 1 0 1 0 1

a a' b' b c c' $\boxed{R}$

reject

LO10. Do the following for the NFA N whose state diagram is shown below.

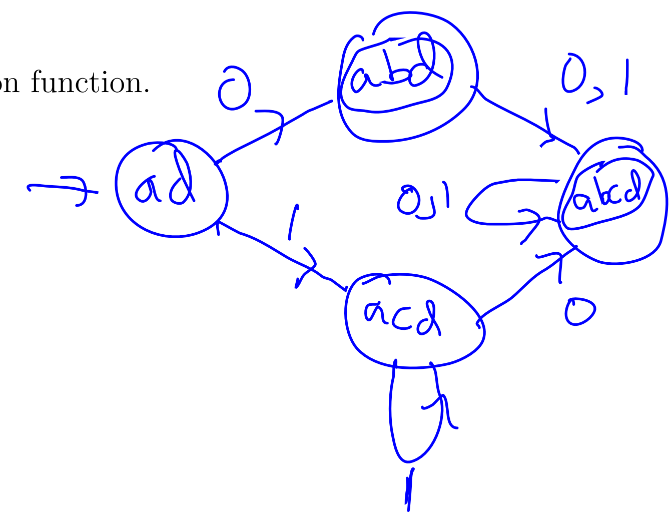


δ:

(a) Provide a table that represents N's δ transition function.

Solution.

$Q \backslash \Sigma$	0	1
a	abd	cd
b	cd	abd
c	acd	$\emptyset$
d	d	acd



(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

Solution.

See Above

(c) Show the computation of M on input $w = 00011$. Is it rejecting or accepting?

Solution.

$\overset{0}{ad}$ $\overset{0}{abd}$ $\overset{0}{abcd}$ $\overset{1}{abcd}$ $\overset{1}{abcd}$ abcd

accept

Since $b \in \{a, b, c, d\}$

LO11. Provide a regular expression E that represents the set of all words over the alphabet $\Sigma = \{a, b\}$, except for "bab" and "ab".

Solution.

$\{\varepsilon, a, b, aa, ba, bb; aaa, aab, aba, abb, baa, bba, bbb\} \cup \{a, b\}^4 \{a, b\}^*$