CECS 329, Learning Outcome Assessment 10, April 25th, Spring 2024, Dr. Ebert

Problems

LO6. Answer/solve the following.

(a) Compute the Gödel number of the program $P = J(2, 1, 2), T(3, 2), Z(5), S(4)$.

(b) A universal program P_U is simulating a program that has 56 instructions and whose Gödel number is

$$x = 2^{31} + 2^{78} + 2^{102} + 2^{182} + 2^{188} + \cdots + 2^{c_{56}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^2 + 2^4 + 2^9 + 2^{13} + 2^{16} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO7. Answer the following.

(a) Describe what it means to be a positive instance of the `Self Accept` decision problem.

(b) In applying the diagonalization method towards proving the undecidability of `Self Accept`, we defined a computable function $g(x)$ in terms of $f(x)$, where $f(x)$ is the decision function for `Self Accept`, which we assume to be total computable. Provide the formula for computing $g(x)$ and describe what needs to be established about $g(x)$ in order for the diagonalization method to be successful.

(c) Suppose URM program P_{121} computes $\phi_{121}(x)$ and that

$$\phi_{121}(x) = \begin{cases} \lfloor\sqrt{x}\rfloor & \text{if } x \text{ is a perfect square} \\ \uparrow & \text{otherwise} \end{cases}$$

What is the value of $g(121)$ and how does this prove that $g \neq \phi_{121}$? Defend your answer.

LO8. An instance of the decision problem `Finite Domain` is a Gödel number x, and the problem is to decide if program P_x has a finite domain set, meaning that the inputs it halts on forms a finite set. Note that $\emptyset$ is considered a finite set. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } P_x \text{ has finite domain} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where e_1 is the Gödel number of the program that computes the function $\phi_{e_1}(x) = 7$.

ii. $x = e_2$, where e_2 is the Gödel number of the program that computes the function

$$\phi_{e_2}(x) = \begin{cases} 1 & \text{if } x \leq 500 \\ \uparrow & \text{otherwise} \end{cases}$$

iii. $x = e_3$, where e_3 is the Gödel number of the program $P = S(1), T(2,1), J(1,2,1)$.

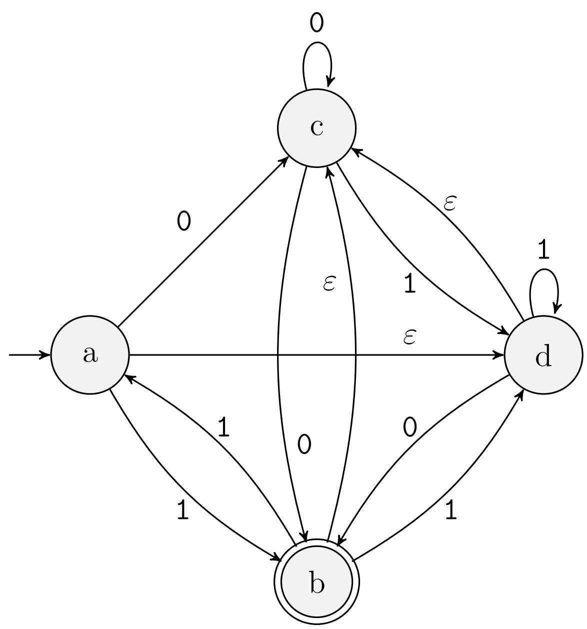
(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not P_x has a finite domain. Do this by writing a program P that uses g and makes use of the `self` programming construct. Then use a proof by cases to show that P creates a contradiction.

LO9. Solve the following.

(a) Provide the state diagram for a DFA M that accepts a binary word w iff either w has an odd number of 1's or an even number of 0's (or both). Yes, zero is an even number.

(b) Show the computation of M on inputs i) $w_1 = 10110111$ and ii) $w_2 = 0110110$.

LO10. Do the following for the NFA N whose state diagram is shown below.



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

(c) Show the computation of M on input $w = 11001$.