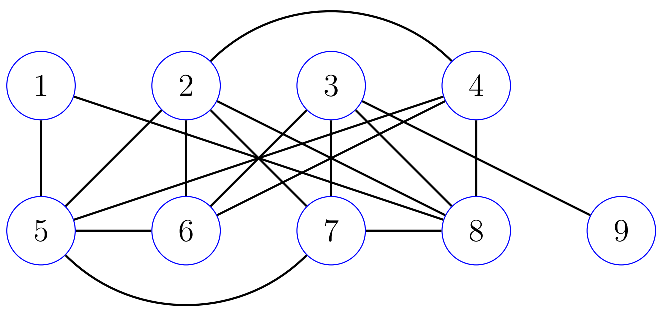
CECS 329, LO Pre-Final Assessment, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to **AT MOST 6 PROBLEMS**. Please use **BOTH SIDES** of each answer sheet to save paper. Make sure your name and Class ID are on each answer sheet.

LO1. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) The simple graph $G = (V, E)$ shown below is an instance of the `Clique` problem with $k = 4$. Draw $f(G, k)$, where f is the mapping reduction from `Clique` to `Half Clique` provided in the Mapping Reducibility lecture exercises.



(c) Verify that both G and $f(G, k)$ are either both positive or both negative instances of their respective problems. Justify your answer.

LO2. Recall that the `3-Dimensional Matching (3DM)` decision problem takes as input three sets A, B, and C, each having size n, along with a set S of triples of the form (a, b, c) where $a \in A$, $b \in B$, and $c \in C$. We assume that $|S| = m \geq n$. The problem is to decide if there exists a subset $T \subseteq S$ of n triples for which each member from $A \cup B \cup C$ belongs to exactly one of the triples. Complete the following steps to show that `3DM` is a member of `NP`.

(a) Define a certificate for an instance (A, B, C, S) of `3DM`. Note: letter C is already being used, so use a different letter to represent the certificate.

(b) Provide a semi-formal verifier algorithm that takes as input `3DM` instance (A, B, C, S) and a certificate and returns 1 iff the certificate is valid.

(c) Provide size parameters for `3DM`.

(d) Provide the verifier's running time and defend your answer.

LO3. An instance $\mathcal{C}$ of `3SAT` consists of clauses $c_1 = (x_1, \overline{x}_2, x_3)$, $c_2 = (\overline{x}_2, x_3, x_4)$, $c_3 = (\overline{x}_1, x_2, \overline{x}_4)$, and $c_4 = (\overline{x}_1, \overline{x}_3, x_4)$. Answer the following questions about the mapping reduction $f(\mathcal{C}) = (G, a, b)$ provided in lecture from `3SAT` to `Directed Hamilton Path` and applied to instance $\mathcal{C}$.

(a) Consider the vertices lc_2 and rc_2 that lie in the chain that connects the left and right halves of the x_2-diamond of G. Describe the orientation of the edge between lc_2 and vertex c_2 as well as the orientation of the edge between rc_2 and c_2. Draw a figure with the three vertices and the two edges and explain why you chose the particular orientations.

(b) Which diamond has no edges connected to vertex c_2? Explain.

(c) Given that $\alpha = (x_1 = x_2 = 0, x_3 = 1, x_4 = 0)$ satisfies $\mathcal{C}$, provide the direction (left or right) that the corresponding Hamilton Path will take in each of the four diamonds of G. Hint: moving left means the path will move across the diamond from left to right.

LO4. Answer the following.

(a) Write either P, NP, or co-NP next to each of the following decision problems in terms of which one best characterizes the complexity of the problem. Note: there is only one best answer for each. Two points each.

i. An instance of `Independent Set` is a graph $G = (V, E)$ and a nonnegative integer $k \geq 0$, and the problem is to decide if G has an independent set of size k.

ii. An instance of `Tautology` is a Boolean formula F and the problem is to decide if F evaluates to 1 for *every* Boolean assignment of its variables.

iii. An instance of `Bounded Clique Size` is a graph $G = (V, E)$ and a nonnegative integer $k \geq 0$, and the problem is to decide if every clique of G has a size that does not exceed k.

iv. An instance of `Triangle` is a graph $G = (V, E)$ and the problem is to decide if G has 3-Clique.

(b) What two properties must a decision problem possess in order for it to be `NP`-complete?

(c) An instance of the `Quadratic Residue (QR)` decision problem is a triple (a, c, m) of positive integers, where $a, c \leq m$, and the problem is to decide if there is an $1 \leq x \leq c$ for which $x^2 \equiv a \bmod m$. For a given instance (a, c, m) of `QR`, describe a certificate in relation to (a, c, m).

LO5. Provide the instructions for a URM program that computes the predicate function $f(x, y) = x - y$, where $x - y = 0$ in case $y > x$. For each register that is used by your program, described its purpose in one or more sentences.

LO6. Solve the following problems.

(a) Provide the URM program P whose Gödel number equals

$$2^{30} + 2^{74} + 2^{112} + 2^{137} - 1.$$

Show all work.

(b) Let P be a URM program that accepts two inputs x and y. What does it mean for P to be universal?

(c) A universal program P_U is simulating a program that has 113 instructions and whose Gödel number is

$$x = 2^3 + 2^{179} + 2^{191} + 2^{196} + 2^{224} + 2^{268} + \cdots + 2^{c_{113}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^5 + 2^{14} + 2^{16} + 2^{23} + 2^{28} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO7. Answer the following. Note: correctly answering two of the three constitutes a pass.

(a) Describe what it means to be a positive instance of the `Self Accept` decision problem.

(b) In applying the diagonalization method towards proving the undecidability of `Self Accept`, we defined a computable function $g(x)$ in terms of $f(x)$, the decision function for `Self Accept`, which we assume to be total computable. Provide the formula for computing $g(x)$ and describe what needs to be established about $g(x)$ in order for the diagonalization method to be successful.

(c) Suppose URM program P_{86} computes $\phi_{86}(x)$ and that

$$\phi_{86}(x) = \begin{cases} \lfloor x/5 \rfloor & \text{if } x \text{ is divisible by 5} \\ \uparrow & \text{otherwise} \end{cases}$$

What is the value of $g(86)$? Show that $g \neq \phi_{86}$.

LO8. An instance of the decision problem `ONTO` is a Gödel number x, and the problem is to decide if function ϕ_x is an onto function, meaning that its range is equal to $\mathcal{N}$. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } \phi_x \text{ is onto} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where e_1 is the Gödel number of the program that computes the function $\phi_{e_1}(y) = 2y$.

ii. $x = e_2$, where e_2 is the Gödel number of the program that computes the function $\phi_{e_2}(y) = 2y^2$.

iii. $x = e_3$, where e_3 is the Gödel number of the program that computes the function $\phi_{e_3}(y) = y$.

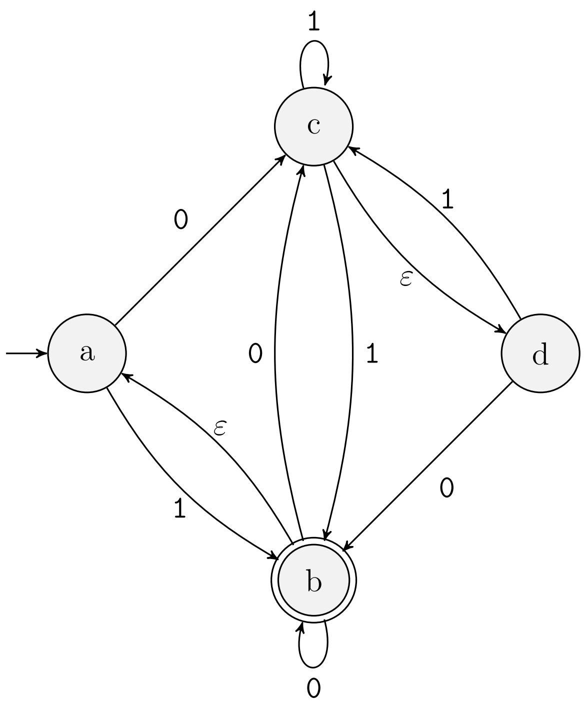
(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not ϕ_x onto. Do this by writing a program P that uses g and makes use of the `self` programming concept. Then explain how P creates a contradiction.

LO9. Solve the following.

(a) Provide the state diagram for a DFA M that accepts all words that contain *exactly* two 0's and *at least* two 1's.

(b) Show the computation of M on inputs i) $w_1 = 010111$ and ii) $w_2 = 01010$.

LO10. Do the following for the NFA N whose state diagram is shown below.



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

(c) Show the computation of M on input $w = 11001$.

LO11. Provide a regular expression that represents the language of all words that contain *exactly* two 0's and *at least* two 1's.