CECS 528, Learning Outcome Assessment 7b, March 17th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

LO3. Answer the following with regards to a correctness-proof outline for the Fractional Knapsack algorithm.

(a) Assume $x_1, x_2, \ldots, x_n$ is an ordering of the items in *decreasing* order of profit density (i.e. profit per unit weight). Let f_i denote the fraction of item i that the FK-algorithm adds to the knapsack. Which of the following is a possible sequence for f_1, f_2, f_3, f_4, f_5?

i. 1,1,0.5,0.3
ii. 1,1,0.4
iii. 0,0,0.3,1,1
iv. 0.25,1,1

(b) Let $f'_1, f'_2, \ldots, f'_n$ be a sequence of fractions that optimizes total profit, and assume that $f_i = f'_i$, for all $i < k$, but $f_k \neq f'_k$. Explain why, in this case, it must be true that $f'_k < f_k$. Hint: in relation to the FKA algorithm, what is the contradiction in case the opposite was true?

(c) For the scenario described in part b, if $f_i = f'_i$, for all $i < k$, but $f_k \neq f'_k$, then what is the relationship between d_k the profit density of x_k, and d_{k+1}, the profit density of x_{k+1}? **Explain.**

LO4. Provide an example of a uniform divide-and-conquer recurrence for an integer function $T(n)$ that forces $T(n) = \Theta(n^3 \log n)$. Justify your answer.

LO5. When applying the linear-time `Merge` algorithm used by Mergesort to the two sorted arrays $a = (3, 7, 11, 15, 17)$ and $b = (1, 2, 4, 8, 16)$, list each pair of numbers that must be compared. Hint: $(3, 1)$ is the first pair. In general, if a and b both have size n, then how many pairs of numbers must be compared?

Solution. The pairs are $(3, 1)$, $(3, 2)$, $(3, 4)$, $(7, 4)$, $(7, 8)$, $(11, 8)$, $(11, 16)$, $(15, 16)$, and $(17, 16)$. In general, there will be $2n - 1$ comparisons, when merging two arrays, each with size n.

LO6. Recall that the `find_statistic` algorithm makes use of Quicksort's partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9 \geq n/4$$

members of a on both its left and right sides, assuming $n \geq 200$.

(a) Rewrite all three inequalities/equalities with updated constants, assuming that the algorithm now uses groups of 11 instead of groups of 5. Also, provide a valid replacement for the inequality $n \geq 200$. Show all work.

Solution.

$$6(\lfloor \frac{1}{2} \lceil \frac{n}{11} \rceil \rfloor - 2) \geq 6(\frac{1}{2} \cdot \frac{n}{11} - 3) = \frac{6n}{22} - 18 \geq n/4$$

which is true iff $n \geq 792$.

(b) Give the rationale for how you decided to replace the 3 on the left side of the very first inequality.

Solution. In each group of 11 there are 6 (5 plus the median of the group) members who are $\leq$ the pivot (median of medians) and 6 that are $\geq$ the pivot.

LO7. Solve the following problems.

(a) The dynamic-programming algorithm that solves the `Edit Distance` optimization problem defines a recurrence for the function $d(i, j)$. In words, what does $d(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $d(i, j)$.

(c) Apply the recurrence from Part b to the words u = aabbab and v = bababa. Show the matrix of subproblem solutions and use it to provide an optimal solution.