CECS 528, Learning Outcome Assessment 6 (Version a), March 10th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

LO2. Recall the use of the disjoint-set data structure for the purpose of improving the running time of the UTS algorithm. For the set of tasks

Task	a	b	c	d	e	f
Deadline	4	0	1	4	4	4
Profit	60	50	40	30	20	10

For each task, show the M-Tree forest after it has been inserted (or at least has attempted to be inserted in case the scheduling array is full). Notice that the earliest deadline is 0, meaning that the earliest slot in the schedule array has index 0. Hint: to receive credit, your solution should show six different snapshots of the M-Tree forest.

LO3. Answer the following with regards to a correctness-proof outline for the Fractional Knapsack algorithm.

(a) Assume $x_1, x_2, \ldots, x_n$ is an ordering of the items in *decreasing* order of profit density (i.e. profit per unit weight). Let f_i denote the fraction of item i that the FK-algorithm adds to the knapsack. Which of the following is a possible sequence for f_1, f_2, f_3, f_4, f_5?

I. 1,1,0.5,0.3

II. 1,1,0.4

III. 0,0,0.3,1,1

IV. 0.25,1,1

(b) Let $f'_1, f'_2, \ldots, f'_n$ be a sequence of fractions that optimizes total profit, and assume that $f_i = f'_i$, for all $i < k$, but $f_k \neq f'_k$. Explain why, in this case, it must be true that $f'_k < f_k$. Hint: in relation to the FKA algorithm, what is the contradiction in case the opposite was true?

(c) For the scenario described in part b, if $f_i = f'_i$, for all $i < k$, but $f_k \neq f'_k$, then what is the relationship between d_k the profit density of x_k, and d_{k+1}, the profit density of x_{k+1}? **Explain.**

LO4. Given $T_1(n) = 27T_1(n/3) + n^3$, and $T_2(n) = aT_2(n/4) + n^2$ what is the greatest possible value that a can assume, and still have $T_2(n) = o(T_1(n))$? Show work and explain.

LO5. Recall that the `Maximum Subsequence Sum (MSS)` problem (Exercise 34) admits a divide-and-conquer algorithm that, on input integer array a, requires computing the maximum of $\text{MSS}_{\text{left}}(a)$, $\text{MSS}_{\text{right}}(a)$, and $\text{MSS}_{\text{mid}}(a)$. If the array is

$$a = 4, -2, 5, -8, 4, 2, -1, 3, -8, 7$$

then

(a) Provide $\text{MSS}_{\text{left}}(a)$, $\text{MSS}_{\text{right}}(a)$. Hint: no need to show work!

(b) Provide the two arrays of sums, Sum_{left} and $\text{Sum}_{\text{right}}$ that are used to compute $\text{MSS}_{\text{mid}}(a)$. Explain how to compute $\text{MSS}_{\text{mid}}(a)$ from these two arrays.

LO6. Recall that the `find_statistic` algorithm makes use of Quicksort's partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9 \geq n/4$$

members of a on both its left and right sides, assuming $n \geq 200$.

(a) Rewrite all three inequalities/equalities with updated constants, assuming that the algorithm now uses groups of 9 instead of groups of 5. Also, provide a replacement for the inequality $n \geq 200$. Show all work.

(b) Give the rationale for how you decided to replace the 3 on the left side of the very first inequality.