CECS 528, Learning Outcome Assessment 3, Feb 10th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

LO1. Determine the asymptotic growth of the sum

$$e^1 + 2e^2 + \cdots + 3e^3.$$

Show all work and justify your approach.

Solution.

$$e^1 + 2e^2 + \cdots + 3e^3 = \Theta(\int_1^n xe^x \mathrm{dx}).$$

$$\int_1^n xe^x \mathrm{dx} = xe^x|_1^n - \int_1^n e^x \mathrm{dx} = \Theta(ne^n).$$

LO2. Recall the use of the disjoint-set data structure for the purpose of improving the running time of the UTS algorithm. For the set of tasks

Task	a	b	c	d	e	f
Deadline	4	4	4	3	3	2
Profit	60	50	40	30	20	10

For each task, show the M-Tree forest after it has been inserted (or at least has attempted to be inserted in case the scheduling array is full). Notice that the earliest deadline is 0, meaning that the earliest slot in the schedule array has index 0. Hint: to receivec credit, your solution should show six different snapshots of the M-Tree forest (not including the initial state).

LO3. Do the following.

(a) In relation to Dijkstra's algorithm, provide a definition for what it means to be i) an (not necessarily optimal) i-neighboring path from source s to an external vertex v, and ii) the i-neighboring distance $d_i(s, v)$ from source s to external vertex v.

Solution.

An i-neighboring path from source s to an external vertex v is a path that uses exactly one edge (the final edge of the path) that is *not* in DDT_i. The i-neighboring distance $d_i(s, v)$ from source s to external vertex v is

$$d_i(s, v) = \min\{\mathrm{cost}(P) | P \text{ is an i-neighboring path from } s \text{ to } v\}.$$

(b) Using the definitions from part a, describe the greedy choice that is made in each round of Dijkstra's algorithm.

Solution. Dijkstra selects the next vertex v to add to the DDT to be that vertex which has the least i-neighboring distance from s. It also adds the external edge of the min-cost i-neighboring path from s to v.

(c) Every path P from s to v must possess an i-neighboring subpath since v is external to DDT_i. Furthermore, by definition of $d_i(s, v)$ and the fact that v has the least i-neighboring distance from s amongst all external vertices, it follows that

$$\text{cost}(P) \geq d_i(s, v)$$

which implies that $d_i(s, v) = d(s, v)$.

LO2 Solution

insert(a,4):

insert(b,4):

insert(c,4):

insert(d,3):

insert(e,3):

insert(f,2):