CECS 528, Learning Outcome Assessment 11b, April 28th, Spring 2023, Dr. Ebert

Problems

LO7. Answer the following.

(a) The Floyd-Warshall algorithm establishes a recurrence for d_{ij}^k. In words, what does d_{ij}^k equal?

(b) Provide the dynamic-programming recurrence d_{ij}^k.

(c) When executing the Floyd-Warshall algorithm, assume

$$d^3 = \begin{pmatrix} 0 & 12 & 14 & 2 & 2 & 6 \\ 9 & 0 & 20 & 13 & 1 & 3 \\ 7 & 5 & 0 & 7 & 6 & 1 \\ 15 & 10 & 19 & 0 & 5 & 2 \\ 9 & 3 & 5 & 6 & 0 & 3 \\ 6 & 5 & 4 & 8 & 2 & 0 \end{pmatrix}$$

has been computed. Use this matrix to compute d^5. Then use d^5 to compute d^6.

LO8. Answer the following.

(a) Provide the dynamic-programming recurrence for computing $\text{mc}(u, v)$ the maximum-cost of any path from vertex u to vertex v in a directed acyclic graph (DAG) $G = (V, E, c)$, where $c(e)$ gives the cost of edge e, for each $e \in E$. Hint: **credit will not be awarded for using $d(u, v)$ instead of mc(u, v).**

(b) Draw the vertices of the following DAG G in a linear left-to-right manner so that the vertices are topologically sorted, meaning, if (u, v) is an edge of G, then u appears to the left of v. The vertices of G are a-h, while the weighted edges of G are

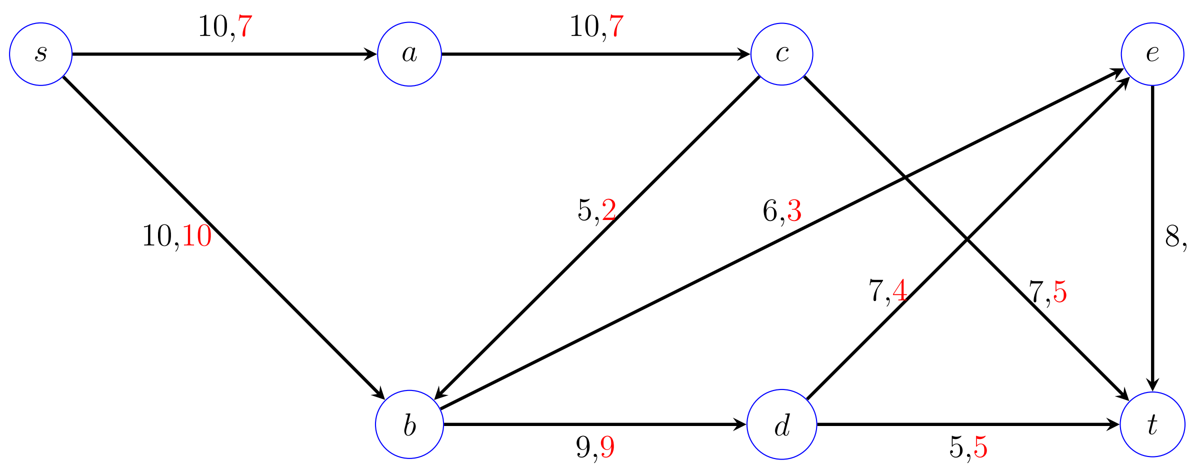
$$(a, b, 18), (a, e, 14), (a, f, 19), (b, c, 13), (b, g, 9), (c, d, 8), (c, g, 13), (c, h, 11), (d, h, 15), (e, b, 5)$$

$$(e, f, 1), (f, b, 19), (f, c, 9), (f, g, 8), (g, d, 4), (g, h, 18).$$

(c) Starting with $u = h$, and working backwards (from right to left in the topological sort), use the recurrence from part a to compute $\text{mc}(u, h)$ for each $u \in \{a, b, \ldots, h\}$, where the ultimate goal is to compute $d(a, h)$.

LO9. A flow f (2nd value on each edge) has been placed in the network G below.

(a) Draw the residual network G_f and use it to determine an augmenting path P. Highlight path P in the network so that it is clearly visible.



(b) In the original network, cross out any flow value that changed, and replace it with its updated value from $f_2 = \Delta(f, P)$.

(c) What one query can be made to a **Reachability** oracle to determine if f_2 is a maximum flow for G? Hint: three inputs are needed for the `reachable` query function. Clearly define each of them.

$\boxed{t = 111}$ $\boxed{M - t = 83}$ $\boxed{28 = J}$

LO10. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

Note: $A = \{22, 44, 45\}$ $M = \sum_{s \in S} s = 194$

(b) For the mapping reduction f : `Subset Sum` $\rightarrow$ `Set Partition`, determine $f(S,t)$ for `Subset Sum` instance ($S = \{4, 7, 15, 19, 22, 38, 44, 45\}, t = 111$). Show work. $f(S,t) = S \cup \{28\}$

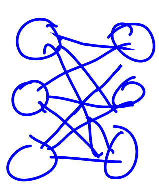
(c) Verify that both (S,t) and $f(S,t)$ are either both positive instances, or are both negative ones. Explain and show work. A above verifies (S,t) is positive

$f(S,t)$ is positive since $A' = \{22, 44, 45\}$, $B' = \{4, 7, 15, 19, 38, 28\}$

$\sum_{a \in A'} a = \sum_{b \in B'} b = 111$

LO11. Recall the mapping reduction $f(\mathcal{C}) = (G, k)$, where f maps an instance of `3SAT` to an instance of the `Clique` decision problem. Given `3SAT` instance

$$\mathcal{C} = \{\underset{C_1}{(x_1, \overline{x}_2, x_5)}, \underset{C_2}{(x_2, x_3, \overline{x}_4)}, \underset{C_3}{(x_1, x_2, x_4)}, \underset{C_4}{(\overline{x}_1, \overline{x}_3, \overline{x}_5)}\}$$



$K_{3,3}$ has 9 edges

answer the following questions about $f(\mathcal{C})$. Hint: to answer these questions you do *not* need to draw G.

(a) How many vertices does G have? Justify your answer. $|V| = (4)(3) = 12$

(b) How many edges does G have? Show work and justify your answer.

(c) Is (G, k) a positive instance of `Clique`? Why or why not? If yes, what size clique must it have? Justify your answer.

1-2: 8
1-3: 8
1-4: 7

2-4: 8
2-3: 8
3-4: 8

$\therefore |E| = (8)(5) + 7 = \boxed{47}$

Yes, since $\alpha = (1, 0, 1, 1, 0)$ satisfies $\mathcal{C}$ and so (G, k) is a positive instance with $k = 4$ and a clique equal to $\{x_1, x_3, x_1, \overline{x}_5\}$