CECS 528, Learning Outcome Assessment 11a, April 28th, Spring 2023, Dr. Ebert

Problems

LO7. Answer/Solve the following questions/problems.

(a) The dynamic-programming algorithm that solves the `Optimal Binary Search Tree` optimization problem defines a recurrence for the function $\text{wac}(i, j)$. In words, what does $\text{wac}(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $\text{wac}(i, j)$.

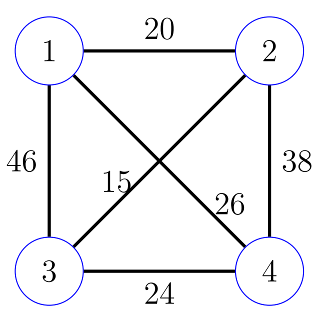
(c) Apply the recurrence from Part b to the keys 1-5 having respective weights 25,45,20,20,30. Show the matrix of subproblem solutions and use it to provide an optimal binary search tree.

LO8. Answer/Solve the following questions/problems.

(a) The dynamic-programming algorithm that solves the `Runaway Traveling Salesperson` optimization problem (Exercise 30 from the Dynamic Programming Lecture) defines a recurrence for the function $\text{mc}(i, A)$. In words, what does $\text{mc}(i, A)$ equal? Hint: do *not* write the recurrence (see Part b). Note: we call it "Runaway TSP" because the salesperson does *not* return to home after visiting each city.

(b) Provide the dynamic-programming recurrence for $\text{mc}(i, A)$.

(c) Apply the recurrence from Part b to the graph below. Show all the necessary computations.



LO9. Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (x_1, \overline{x}_3), (x_1, \overline{x}_4), (\overline{x}_1, x_3), (x_2, x_4), (\overline{x}_2, x_3)(\overline{x}_3, \overline{x}_4)\}.$$

(a) Draw the implication graph $G_{\mathcal{C}}$.

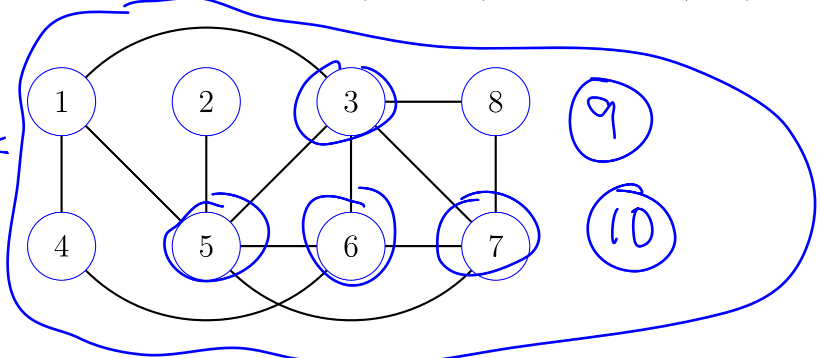
(b) Find a literal l for which i) R_l is an inconsistent reachability set, ii) $R_{\overline{l}}$ is a consistent reachability set, and iii) $\alpha_{R_{\overline{l}}}$ satisfies *all* the clauses of $\mathcal{C}$. For full credit clearly state the literal l you have chosen and verify that each of the three properties are satisfied. Hint: for example, if you choose $l = \overline{x}_3$, then $\overline{l} = \overline{\overline{x}}_3 = x_3$.

LO10. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Clique` $\rightarrow$ `Half Clique` provided in Exercises 11 and 12, and for `Clique` instance $(G, k = 5)$, determine $f(G, k)$, where G is shown below.

$f(G,k) =$



(c) Verify that (G, k) and $f(G, k)$ are either both positive instances or both negative instances of their respective decision problems. Defend your answer. Both are negative since the largest clique is $\{3,5,6,7\}$ has size $= 4$

x_1

LO11. Recall the mapping reduction $f(\mathcal{C}) = (S, t)$, where f maps an instance of `3SAT` to an instance of the `Subset` decision problem. Given `3SAT` instance

y_1 : 1 0 1 0

z_1 : 0 0 0 1

$\overline{x}_1$

$n = 5$ variables

$m = 4$ clauses

$$\mathcal{C} = \{\underbrace{(x_1, \overline{x}_2, x_5)}_{c_1}, \underbrace{(x_2, x_3, \overline{x}_4)}_{c_2}, \underbrace{(x_1, x_2, x_4)}_{c_3}, \underbrace{(\overline{x}_1, \overline{x}_3, \overline{x}_5)}_{c_4}\}$$

answer the following questions about $f(\mathcal{C})$. Hint: to answer these questions you are *not* required to draw the table.

(a) What is the value of t? $t = 11{,}113{,}333$

(b) How many numbers (counting repeats) are in S? What is the largest (in terms of numerical value) number in S? $|S| = (2)(5) + (2)(4) = 18$,

(c) Use satisfying assignment $\alpha = (x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0, x_5 = 0)$ for $\mathcal{C}$ and use it to identify a subset A of S that sums to t. List all the members of A.

largest : $y_1 = 100{,}001{,}010$

$A = \{y_1, y_2, z_3, z_4, z_5, g_1, h_1, g_2, g_3, g_4\}$