CECS 528, Learning Outcome Assessment 10b, April 21st, Spring 2023, Dr. Ebert

Problems

LO6. Given that $r = ae + bg$, $s = af + bh$, $t = ce + dg$, and $u = cf + dh$ are the four entries of AB, and Strassen's products are obtained from matrices

$$A_1 = a, B_1 = f - h, A_2 = a + b, B_2 = h, A_3 = c + d, B_3 = e, A_4 = d, B_4 = g - e,$$

$$A_5 = a + d, B_5 = e + h, A_6 = b - d, B_6 = g + h, A_7 = a - c, B_7 = e + f,$$

Compute $P_1, \ldots, P_7$ and use them to compute r, s, t, and u.

LO7. Solve the following problems.

(a) The dynamic-programming algorithm that solves the `Optimal Binary Search Tree` optimization problem defines a recurrence for the function $\text{wac}(i, j)$. In words, what does $\text{wac}(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $\text{wac}(i, j)$.

(c) Apply the recurrence from Part b to the keys 1-5 having respective weights 30,45,20,25,10. Show the matrix of subproblem solutions and use it to provide an optimal binary search tree.

LO8. Solve the following problems.

(a) The dynamic-programming algorithm that solves the `Longest Common Subsequence (LCS)` optimization problem defines a recurrence for the function $\text{lcs}(i, j)$. In words, what does $\text{lcs}(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $\text{lcs}(i, j)$.

(c) Apply the recurrence from Part b to the words $u =$ abaabb and $v =$ aabbba. Show the matrix of subproblem solutions and use it to provide an optimal solution.

LO9. Part a refers to the original `2SAT` algorithm that makes oracle queries, while Part b refers to the improved `2SAT` algorithm.

(a) Suppose you have been given an unsatisfiable instance $\mathcal{C}$ of `2SAT` that consists of 300 variables and 500 binary clauses. If you apply the original `2SAT` algorithm to $\mathcal{C}$, what is the *worst case* number of oracle queries that will have be made before concluding that $\mathcal{C}$ is unsatisfiable? Explain.

(b) Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_3), (\overline{x}_1, x_2), (\overline{x}_1, x_3), (\overline{x}_1, x_4), (x_2, x_3), (\overline{x}_2, \overline{x}_4), (\overline{x}_3, x_4)\}.$$

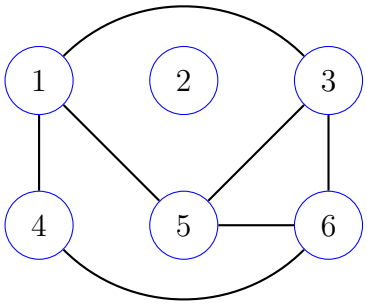
i. Draw the implication graph $G_{\mathcal{C}}$.

ii. Find a literal l for which i) R_l is an inconsistent reachability set, ii) $R_{\overline{l}}$ is a consistent reachability set, and iii) $\alpha_{R_{\overline{l}}}$ satisfies *all* the clauses of $\mathcal{C}$. For full credit clearly state the literal l you have chosen and verify that each of the three properties are satisfied. Hint: for example, if you choose $l = \overline{x}_3$, then $\overline{l} = \overline{\overline{x}}_3 = x_3$.

LO10. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Max Independent Set` $\rightarrow$ `Clique` provided in leccture, and for `MIS` instance G, determine $f(G)$, where G is shown below.



(c) Verify that the (numerical) answer to both G and $f(G)$ are identical. Explain.