**CECS 528, Learning Outcome Assessment 10a, April 21st, Spring 2023, Dr. Ebert**

# Problems

LO6. Recall that the `find_statistic` algorithm makes use of Quicksort's partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9.$$

members of array $a$ on both its left and right sides. Disect the expression to the left of $\geq$ by explaining the significance of each of the following. Three out of four correct for passing.

(a) $\frac{n}{5}$

(b) 3

(c) $\frac{1}{2}$

(d) -2

LO7. Answer the following.

(a) The Floyd-Warshall algorithm establishes a recurrence for $d_{ij}^k$. In words, what does $d_{ij}^k$ equal?

(b) Provide the dynamic-programming recurrence $d_{ij}^k$.

(c) When executing the Floyd-Warshall algorithm, assume

$$d^4 = \begin{pmatrix} 0 & 12 & 14 & 2 & 2 & 6 \\ 9 & 0 & 20 & 13 & 1 & 3 \\ 7 & 5 & 0 & 7 & 6 & 1 \\ 15 & 10 & 19 & 0 & 5 & 2 \\ 9 & 3 & 5 & 6 & 0 & 3 \\ 6 & 5 & 4 & 8 & 2 & 0 \end{pmatrix}$$

has been computed. Use this matrix to compute $d^5$. Then use $d^5$ to compute $d^6$.

LO8. Answer the following.

(a) Provide the dynamic-programming recurrence for computing $d(u, v)$ the distance from vertex $u$ to vertex $v$ in a directed acyclic graph (DAG) $G = (V, E, c)$, where $c(e)$ gives the cost of edge $e$, for each $e \in E$.

(b) Draw the vertices of the following DAG $G$ in a linear left-to-right manner so that the vertices are topologically sorted, meaning, if $(u, v)$ is an edge of $G$, then $u$ appears to the left of $v$. The vertices of $G$ are a-h, while the weighted edges of $G$ are

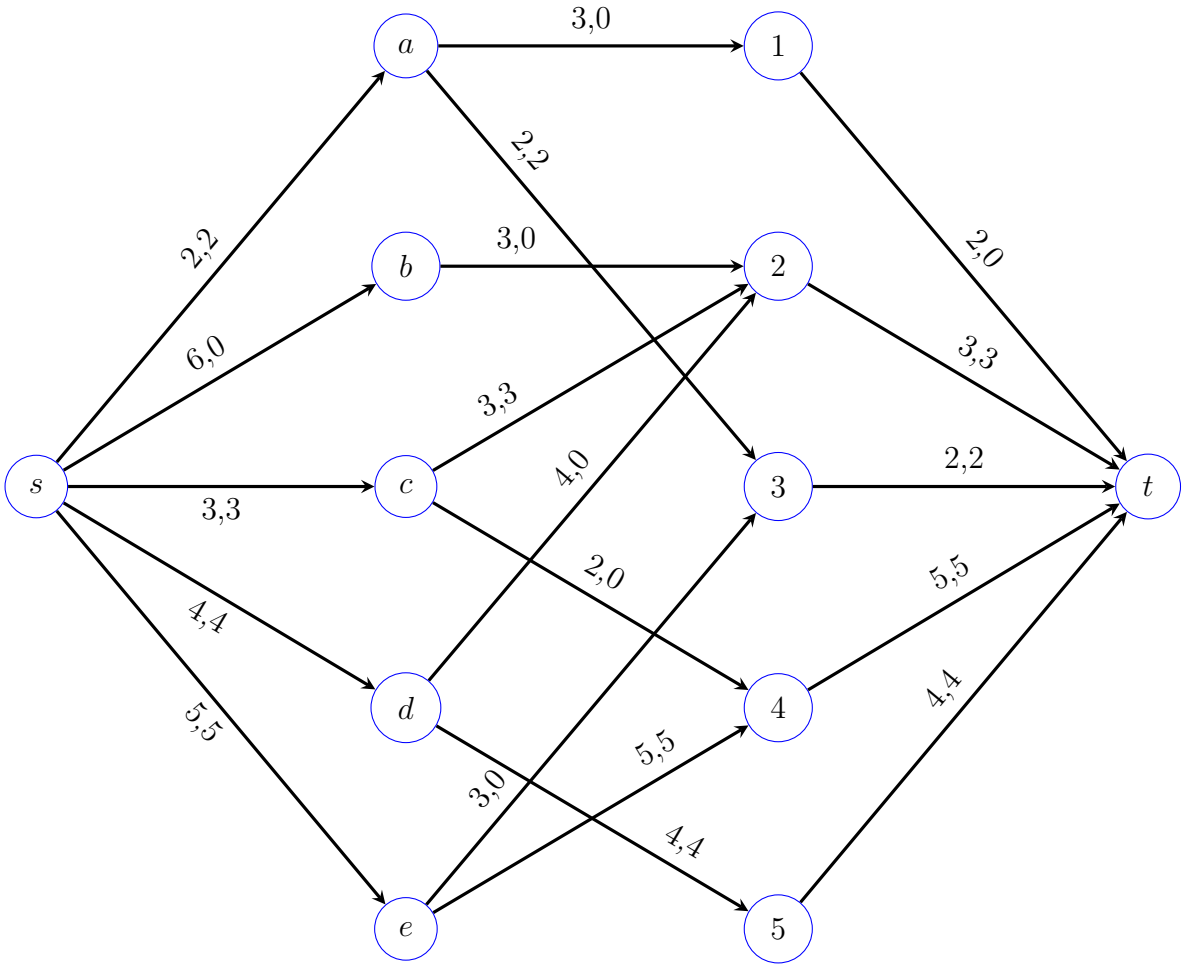
$$(a, b, 18), (a, e, 14), (a, f, 19), (b, c, 13), (b, g, 9), (c, d, 8), (c, g, 13), (c, h, 11), (d, h, 15), (e, b, 5)$$

$$(e, f, 1), (f, b, 19), (f, c, 9), (f, g, 8), (g, d, 4), (g, h, 18).$$

(c) Starting with $u = h$, and working backwards (from right to left in the topological sort), use the recurrence from part a to compute each of $d(u, h)$, where the ultimate goal is to compute $d(a, h)$.

LO9. A flow $f$ (in red) has been placed in the network $G$ below.

(a) Draw the residual network $G_f$ and use it to determine an augmenting path $P$. Highlight path $P$ in the network so that it is clearly visible.




(b) Redraw the original network, but with the $f$ flow values being replaced by the $\Delta(f, P)$ flow values.

(c) What one query is needed to the Reachability-oracle in order to determine if $f_2 = \Delta(f, P)$ is a maximum flow for $G$?

LO10. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem $A$ to decision problem $B$. (See Page 4 of Turing and Map Reducibility Lecture).

(b) For the mapping reduction $f$ : `Subset Sum` $\rightarrow$ `Set Partition`, determine $f(S, t)$ for `Subset Sum` instance $(S = \{2, 7, 10, 14, 31, 33, 38, 46\}, t = 71)$. Show work.

(c) Verify that the (yes/no) answer to both $(S, t)$ and $f(S, t)$ are identical. Explain.