CECS 329, Final Exam, May 11th, Spring 2023, Dr. Ebert

Rules for Completing the Problems

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION allowed when solving these problems. Make sure all these items are put away BEFORE looking at the problems. FAILURE TO ABIDE BY THESE RULES MAY RESULT IN A FINAL COURSE GRADE OF F.

Directions

Choose up to **six problems** to solve. Clearly mark which problems you want us to grade by placing X's in the middle "Grade?" row. **If you don't mark any problems for us to grade or mark 7 or more problems, then we will record grades for the 6 that received the *fewest* points.**

Problem	1	2	3	4	5	6
Grade?						
Points/P/NP						

Problem	LO1	LO2	LO3	LO4	LO5	LO6	LO7	LO8	LO9
Grade?									
P/NP									

1. Solve/answer the following.

   a. Formally state Kleene's 2nd Recursion Theorem. Clearly define each of the symbols used in the statement. (10 pts)

   b. Explain the significance of Kleene's 2nd Recursion Theorem in relation to computer programming. (10 pts)

   c. The proof of Kleene's 2nd Recursion Theorem involves defining a URM program $P = ABC$, where A, B, and C are subprograms that are concatenated together to form P. Explain the role played by each subprogram. Please include information on the effect each has on the machine register(s). (30 pts)

      A.

      B.

      C.

2. Answer the following. Note: correctly solving this problem counts for passing LO10.

   a. Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B. (10 pts)

   b. Is $f(n) = n^3 + 7$ a valid mapping reduction from the `Even` decision problem to the `Odd` decision problem? Justify your answer. (15 pts)

3. Answer the following.

   a. Provide the definition of what it means for decision problem A to be *Turing reducible* to decision problem B. (10 pts)

   b. Suppose $A \le_T B$ via the logical formula

   $$x \in A \Leftrightarrow (\text{query}_B(x^2) \wedge \text{query}_B(x^2 + 3x)) \vee (x \ge 100),$$

   and $B \le_T C$ via the logical formula

   $$x \in B \Leftrightarrow (\text{query}_C(3x + 2) \vee (x \le 75)) \wedge \text{query}_C(5x^3).$$

   Prove a logical formula that establishes the fact that $A \le_T C$. Hint: e.g. $\text{query}_B(x^2)$ is a query to the B-oracle regarding whether or not $x^2 \in B$. (15 pts)

4. Answer the following. Note: correctly solving at least three parts counts for passing LO11.

   a. Recall the mapping reduction $f(\mathcal{C}) = (G, k)$, where f maps an instance of `3SAT` to an instance of the `Clique` decision problem. Given `3SAT` instance

   $$\mathcal{C} = \{(x_1, x_2, x_4), (\overline{x}_2, x_3, \overline{x}_4), (\overline{x}_1, x_2, x_3), (\overline{x}_1, \overline{x}_3, \overline{x}_4),$$

   how many vertices does G have? Explain. (5 pts)

   b. Continuing from part a, what is the value of k? Does G have a k-clique? Defend you answer and provide a k-clique if one exists. (10 pts)

   c. Recall the mapping reduction $f(\mathcal{C}) = (G, a, b)$, where f maps an instance of `3SAT` to an instance of the `DHP` decision problem. Given `3SAT` instance

   $$\mathcal{C} = \{c_1 = (x_1, x_2, x_4), c_2 = (\overline{x}_2, x_3, \overline{x}_4), c_3 = (\overline{x}_1, x_2, x_3), c_4 = (\overline{x}_1, \overline{x}_3, \overline{x}_4)\}$$

   how many "island" vertices does G have? Explain. (5 pts)

   d. Continuing from part c, consider the three vertices lc_3, rc_3, and c_3, where lc_3 and rc_3 belong to the x_2-diamond. Show how these three vertices are connected to each other. Hint: a variable being set to 0 is analogous to moving from left to right through the diamond. (10 pts)

5. Circle the correct answer (8 pts each) for each multiple choice question. Note: correctly answering at least two counts for passing LO12.

   a. Which of the following decision problems are most likely in class `P`?

      i. An instance of `Triangle` is a simple graph G and the problem is to decide if G has a 3-clique.
      ii. `3SAT`.
      iii. `DHP`.
      iv. None of the above are likely in class `P`.

   b. Which of the following certificates is best suited for checking if some instance G of the `Hamilton Cycle` decision problem is a positive instance.

      i. Certificate C is an ordering of the vertices of G. For example, if G's vertices are $\{a, b, c, d\}$, then dbca is one possible certificate.
      ii. Certificate C is a subset of k vertices of G for some $k \geq 0$.
      iii. Certificate C is an ordering (defined above) of the edges of G.
      iv. Certificate C is a subset of k edges of G for some $k \geq 0$.

   c. If we know that $A \leq_m^p B$ and A is *not* in `P`, then it must be true that

      i. A is `NP`-complete.
      ii. B is not in `P`.
      iii. B is in `NP`.
      iv. None of the above are necessarily true.

6. Given a simple graph $G = (V, E)$, a **dominating set** for G is a set of vertices $D \subseteq V$ for which every vertex $v \in V$ is either a member of D, or is adjacent to some vertex in D.

   a. If C is a vertex cover for G, explain why it is also a dominating set for G. (5 pts)

   b. Give an example of a graph G that has a dominating set D, but D is *not* a vertex cover. Defend your example. (5 pts)

   c. Describe a mapping reduction f : `Vertex Cover` $\rightarrow$ `Dominating Set` from the `Vertex Cover` decision problem to the **Dominating Set** decision problem. For example, an instance of **Dominating Set** is a pair (G, k), where G is a simple graph and $k \geq 0$ is an integer, and the problem is to decide if G has a dominating set of size k. Clearly define $f(G, k)$ and provide an example of your reduction. Hint: make triangles. (15 pts)