**CECS 329, Exam 1, February 16th, Spring 2023, Dr. Ebert**

## Rules for Completing the Problems

**NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION allowed when solving these problems. Make sure all these items are put away BEFORE looking at the problems. FAILURE TO ABIDE BY THESE RULES MAY RESULT IN A FINAL COURSE GRADE OF F**.

## Directions

Choose up to **six problems** to solve. Clearly mark each problem you want graded by placing an 'X' or check mark in the appropriate box. **If you don't mark any problems for us to grade or mark 7 or more problems, then we will record grades for the 6 that received the *fewest* points.**

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | LO1 |
|---|---|---|---|---|---|---|---|
| Grade? | | | | | | | |
| Result | | | | | | | |

Your Full Name:

1. Do the following. Note: correctly solving this problem counts for passing LO1.

   a. Provide the state diagram for a DFA that accepts all words $w$ over $\Sigma$ for which the top layer of $w$ minus the middle layer of $w$ equals the bottom layer of $w$, where each layer is viewed as a binary number whose left most bit is the least significant bit, and whose rightmost bit is the most significant bit. For example, consider the two words

$$w_1 = \begin{matrix} 010110 \\ 001010 \\ 011000 \end{matrix}, \qquad \text{and} \qquad w_2 = \begin{matrix} 11001 \\ 10100 \\ 00110 \end{matrix}.$$

   The DFA should accept $w_1$ since the top layer represents the number 26, the middle layer 20, the bottom layer 6, and $26 - 20 = 6$. However, it should *not* accept $w_2$ since the top layer represents the number 19, the middle layer 5, the bottom layer 12, and $19 - 5 = 14 \neq 12$. (20 pts) Hint: you can solve this problem with a 3-state DFA.

   b. Show the computation of your DFA for input $w_1$ defined above. (5 pts)

2. Do the following.

   a. Provide the state diagram for an NFA that uses only three states and accepts the binary language $L$ described as follows. Binary word $w \in L$ if either i) $w$ is empty, ii) $w$ consists of all 0's, or iii) each 1 bit in $w$ is next to exactly one other 1 bit. For example, 01100011 and 000 are words in $L$, while 0100110 and 01101110 are *not* words in $L$. (10 pts)

   b. Provide a regular expression $E$ for which $L(E)$ is the language described in part a. Note: correctly solving this part counts for passing LO3. (15 pts)

3. Do the following.

   a. Provide a three-state DFA $M_1$ that accepts all words that contain the subword 01. (5 pts)

   b. Provide a three-state DFA $M_2$ that accepts all words that have a number of 1's that is divisble by three. For example, $M_2$ should accept 01111011, but not 110110. (5 pts)

   c. Use the Intersection Algorithm to provide a nine-state DFA $M_3$ that accepts all words that contain the subword 01 and have a number of 1's that is divisible by three. (15 pts)

4. Do the following.

   a. Use the fact that regular languages are closed under intersection (see Problem 3) and complement to show that they are also closed under union and symmetric difference. (13 pts)

   b. Does the word 0011001101 belong to $\{1101, 10, 001, 0011\}^*$? Explain. (6 pts)

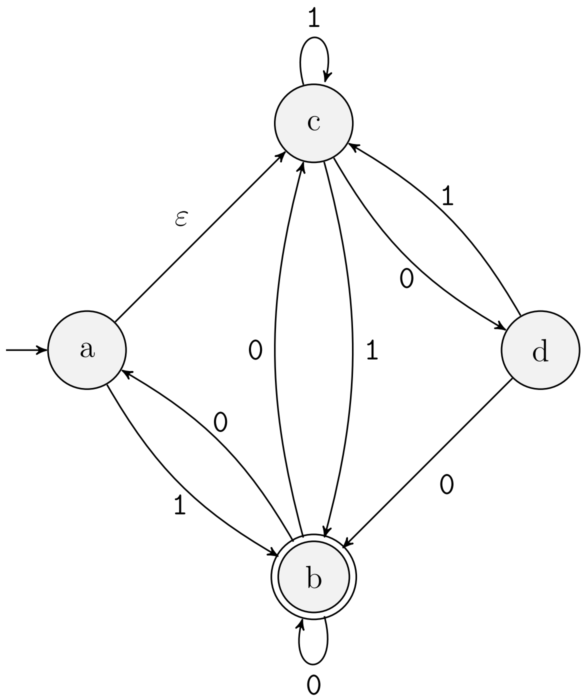
   c. Repeat part b for the word 001110111010011. (6 pts)

5. Do the following.

a. Provide a context-free grammar $G = (V, \Sigma, R, S)$ for which $L(G)$ consists of all words of the form $x\#y$ where i) $x, y \in \{\text{a,b}\}^*$, ii) $x$ has an equal number of a's and b's, and iii) $y$ has any number of a's, and exactly two b's. (15 pts)

b. Use the grammar from part a to provide a leftmost derivation of the word abbaab#abaab. (10 pts)

6. Do the following for the NFA $N$ whose state diagram is shown below. Note: correctly solving this part counts for passing LO2.




a. Provide a table that represents $N$'s $\delta$ transition function.

b. Use the table from part a to convert $N$ to an equivalent DFA $M$ using the method of subset states. Draw $M$'s state diagram.

c. Show the computation of $M$ on input $w = 11000$.

LO1. Do the following.

(a) Provide the state diagram of a DFA $M$ that accepts all binary words that begin and end with the same bit value. For example, 0, 101, and 0100 should be accepted, but $\varepsilon$, 01, and 110 should be rejected.

(b) Show the computation of $M$ on input i) $w = 1101$ and ii) $w = 1110$.