CECS 329, Learning Outcome Assessment 8, April 6th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

LO4. Do the following.

(a) Provide a context free grammar $G = (V, \Sigma, R, S)$ for which $L(G)$ is the set of binary words that have i) a length of at least two, and ii) an even number of bits and for which the two middle bits are 11.

Solution. We have

$$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 11.$$

(b) Use G to provide a leftmost derivation of $w = 01011110$.

Solution. We have

$$S \rightarrow 0S0 \rightarrow 01S10 \rightarrow 010S110 \rightarrow 01011110.$$

LO5. Do the following.

(a) Provide the instructions for a URM that computes the function $f(x) = 3x + 5$.

Solution.

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1:J(1,2,13)
2:S(5) //Store 3 in R5
3:S(5)
4:S(5)
5:S(2) //The goal is to add 3x to R2
6:S(3) //The goal is to add x to R3
7:J(1,3,9) //R3 now holds x
8:J(1,1,5)
9:S(4)
10:J(4,5,13)
11:Z(3) //Prepare to add x to R2 and R3
12:J(1,1,5)
13:S(2)
14:S(2)
15:S(2)
16:S(2)
17:S(2)
18:T(2,1)
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(b) For each register used in your program, use one or more sentences to describe its purpose.

- R1 stores input x.
- R2 stores the program output.
- R3 counts up to x (three times) and resets each time it counts up to x.
- R4 counts how many times R3 has counted up to x. Program continues to run until R4 itself counts up to 3.
- R5 stores the value 3

LO6. Do the following.

(a) Compute the Gödel number for program $P = Z(4), J(2,1,3), T(2,4), S(2)$. Write your answer as a sum of powers of two minus 1.

Solution.

$$\beta(Z(4)) = 4(3) = 12.$$

$$\beta(J(2,1,3)) = 4\xi(1,0,2) + 3 = 4\pi(\pi(1,0),2) + 3 = 4\pi(1,2) + 3 = 4(9) + 3 = 39.$$

$$\beta(T(2,4)) = 4\pi(1,3) + 2 = 4(13) + 2 = 54.$$

$$\beta(S(2)) = 4(1) + 1 = 5.$$

$$\gamma(P) = \tau(12, 39, 54, 3) = 2^{12} + 2^{52} + 2^{107} + 2^{113} - 1.$$

(b) Provide the URM program P whose Gödel number equals

$$2^{13} + 2^{25} + 2^{38} + 2^{53} - 1.$$

Solution. $\gamma^{-1}(x) = S(4), J(1,2,2), Z(4), T(3,1)$.

LO7. Answer and solve the following.

(a) When simulating a program P_x on input y via a universal program P_U, which of the following is a valid reason for why P_U must know the maximum register index used by P_x?

i. **X** The location of the program counter within a configuration directly depends on the maximum register index.

ii. The location of input y within a configuration directly depends on the maximum register index.

iii. The decoding of any instruction of P_x directly depends on the maximum register index.

iv. All of the above are valid reasons for why P_U must know the maximum register index used by P_x.

(b) A universal program P_U is simulating a program that has 100 instructions and whose Gödel number is

$$x = 2^{13} + 2^{18} + 2^{30} + 2^{85} + 2^{c_5} + \cdots + 2^{c_{100}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^2 + 2^{11} + 2^{13} + 2^{14} + 2^{19} - 1,$$

then provide the next configuration of the computation *and* its τ encoding.

LO8. Do the following.

(a) In one or more complete sentences, describe what is asserted by the Church-Turing Thesis.
Solution. See Universal Programs lecture notes.

(b) Consider the function

$$g(x) = \begin{cases} 1 & \text{if } \phi_x \text{ is total} \\ 0 & \text{otherwise} \end{cases}$$

In other words, $g(x) = 1$ iff program P_x halts on all of its inputs. We want to prove that $g(x)$ is undecidable, meaning there is no URM program that computes g. To do this, let's assume that $g(x)$ is computable by some URM program G. Then define the function $f(x)$ as follows.

$$f(x) = \begin{cases} P_x(x) + 1 & \text{if } g(x) = 1 \\ 0 & \text{if } g(x) = 0 \end{cases}$$

(c) Let e denote the Gödel number of the URM program F that computes $f(x)$ from part b. In other words $F = P_e$. Show that a contradiction arises when we try to compute $F(e) = P_e(e)$.