NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

## Problems

LO3. Provide a regular expression that represents the set of binary words that have at least one 0 and at least one 1.

LO4. Do the following.
(a) Provide a context free grammar $G=(V, \Sigma, R, S)$ for which $L(G)$ is the set of binary words that have an odd number of 0 's and exactly one 1 .
(b) Use $G$ to provide a leftmost derivation of $w=000100$.

LO5. Let $\operatorname{Trunc}(x, i)$ denote the number $x$ with its first $i$ digits cut off. For example, $\operatorname{Trunc}(958,0)=$ 958 , $\operatorname{Trunc}(958,1)=95, \operatorname{Trunc}(958,2)=9$, and $\operatorname{Trunc}(958, i)=0$ for every $i \geq 3$. Provide a recursive definition of $\operatorname{Trunc}(x, i)$. You may use any PR functions from the General Models of Computation lecture examples and exercises.

LO6. Do the following.
(a) Compute the Gödel number for program $P=Z(3), J(2,1,2), T(3,1), S(4)$. Write your answer as a sum of powers of two minus 1 (see part b).
(b) Provide the URM program $P$ whose Gödel number equals

$$
2^{9}+2^{36}+2^{56}+2^{77}-1
$$

LO7. Do the following.
(a) Which of the following is not needed by a universal program $P_{U}$ on inputs $x$ and $y$ ?
i. the maximum index of any register used by $P_{x}$
ii. the number of instructions of $P_{x}$
iii. the maximum number of configurations used in the computation of $P_{x}$ on input $y$
iv. All of the above are needed to simulate the computation of $P_{x}$ on input $y$.
(b) Consider the computation of $P_{U}(x, 2)$, where $x=2^{5}+2^{11}+2^{27}+2^{34}-1$. If the current configuration of $P_{x}(2)$ has encoding $\sigma=2^{2}+2^{5}+2^{6}+2^{10}-1$, then provide the next configuration of the computation and its encoding.

