CECS 329, Learning Outcome Assessment 7, March 16th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

LO3. Provide a regular expression that represents the set of binary words that have at least one 0 and at least one 1.

Solution. Key idea: a necessary and sufficient condition is that at least one of the words from $\{01, 10\}$ must be in a word of the language (why?).

$$\{0,1\}^*\{01,10\}\{0,1\}^*.$$

LO4. Do the following.

(a) Provide a context free grammar $G = (V, \Sigma, R, S)$ for which $L(G)$ is the set of binary words that have an odd number of 0's and exactly one 1.

Solution. Idea: X generates an odd number of 0's, while Y generates an even number of 0's. In other words,

$$\text{ODD} = \text{ODD} \oplus \text{EVEN} = \text{EVEN} \oplus \text{ODD}.$$

$$S \to X1Y \mid Y1X$$

$$X \to 00X \mid 0.$$

$$Y \to 00Y \mid \varepsilon.$$

(b) Use G to provide a leftmost derivation of $w = 000100$.

$$S \to X1Y \to 00X1Y \to 0001Y \to 000100Y \to 000100.$$

LO5. Let $\text{Trunc}(x, i)$ denote the number x with its first i digits cut off. For example, $\text{Trunc}(958, 0) = 958$, $\text{Trunc}(958, 1) = 95$, $\text{Trunc}(958, 2) = 9$, and $\text{Trunc}(958, i) = 0$ for every $i \geq 3$. Provide a *recursive* definition of $\text{Trunc}(x, i)$. You may use any PR functions from the General Models of Computation lecture examples and exercises.

Solution. Key idea: to get the next truncated value, divide the current truncated value by 10. For example, $8473/10 = 847$, $847/10 = 84$, $84/10 = 8$, $8/10 = 0$, etc..

Base case: $\text{Trunc}(x, 0) = x$

Recursive case: $\text{Trunc}(x + 1, i) = \text{Trunc}(x, i)/10$.

LO6. Do the following.

(a) Compute the Gödel number for program $P = Z(3), J(2,1,2), T(3,1), S(4)$. Write your answer as a sum of powers of two minus 1 (see part b).
Solution.

$$\beta(Z(3)) = 4(2) = 8.$$

$$\beta(J(2,1,2)) = 4\xi(1,0,1) + 3 = 4\pi(\pi(1,0),1) + 3 = 4\pi(1,1) + 3 = 4(5) + 3 = 23.$$

$$\beta(T(3,1)) = 4\pi(2,0) + 2 = 4(3) + 2 = 14.$$

$$\beta(S(4)) = 4(3) + 1 = 13.$$

$$\gamma(P) = \tau(8,23,14,13) = 2^8 + 2^{32} + 2^{47} + 2^{61} - 1.$$

(b) Provide the URM program P whose Gödel number equals

$$2^9 + 2^{36} + 2^{56} + 2^{77} - 1.$$

Solution.
$P = S(3), T(1,4), J(1,1,3), Z(6)$
For example, to get $J(1,1,3)$, we have $56 - 36 - 1 = 19$ and $19 \bmod 4 = 3$ which implies a Jump instruction. Also, $(19-3)/4 = 4$, and $\pi^{-1}(4) = (0,2)$ since $(4+1) = 5 = 2^0(2(2)+1)$. Finally, π^{-1} of the exponent 0 equals $(0,0)$ since $(0+1) = 1 = 2^0(2(0)+1)$. Therefore, we have $J(0+1, 0+1, 2+1) = J(1,1,3)$.

LO7. Do the following.

(a) Which of the following is *not* needed by a universal program P_U on inputs x and y?

i. the maximum index of any register used by P_x
ii. the number of instructions of P_x
iii. the maximum number of configurations used in the computation of P_x on input y
iv. All of the above are needed to simulate the computation of P_x on input y.

Solution. iii), since in general it is not possible know the maximum number of configurations used in the computation of a program on one of its inputs. In fact, some computations never terminate.

(b) Consider the computation of $P_U(x, 2)$, where $x = 2^5 + 2^{11} + 2^{27} + 2^{42} - 1$. If the current configuration of $P_x(2)$ has encoding $\sigma = 2^2 + 2^5 + 2^6 + 2^{10} - 1$, then provide the next configuration of the computation *and* its encoding.
Solution.
$P_x = S(2), S(2), J(1,2,1), T(3,1)$.
The current configuration is $\tau^{-1}(\sigma) = (2,2,0,3)$. Thus, the program counter equals 3 and the next instruction is $J(1,2,1)$. Finally, since $R_1 = R_2 = 2$, the next configuration is $(2,2,0,1)$ and

$$\tau(2,2,0,1) = 2^2 + 2^5 + 2^6 + 2^8 - 1.$$