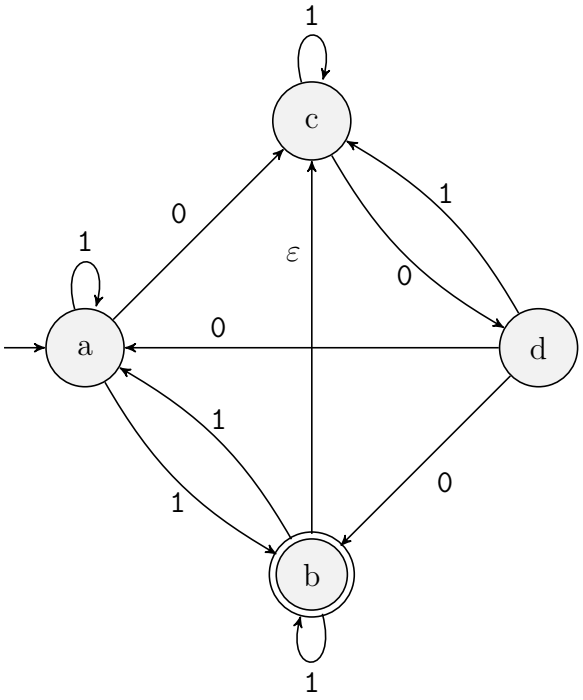
CECS 329, Learning Outcome Assessment 6, March 9th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

LO2. Do the following for the NFA N whose state diagram is shown below.



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

(c) Show the computation of M on input $w = 11001$.

LO3. Provide a regular expression that represents the set of words of the form

$$(w_1\#w_2) + (w_3\#w_4) + \cdots + (w_{2n-1}\#w_{2n}),$$

where each w_i, $i = 1, \ldots, 2n$, is a nonempty binary word. For example $(010\#1) + (11\#110)$ is one such word. Hint: the underlying alphabet is $\Sigma = \{0, 1, (,), \#, +\}$.

LO4. Do the following.

(a) Provide a context free grammar $G = (V, \Sigma, R, S)$ for which $L(G)$ is the set of words of the form

$$(w_1\#w_2) + (w_3\#w_4) + \cdots + (w_{2n-1}\#w_{2n}),$$

where each w_i, $i = 1, \ldots, 2n$, is a nonempty binary word. For example $w = (0\#1)+(10\#0)$ is one such word. Hint 1: the terminal set is $\Sigma = \{0, 1, (,), \#, +\}$. Hint 2: use the start symbol to generate the first left parenthesis, but use a different variable to generate all subsequent left parentheses.

(b) Use G to provide a leftmost derivation of $w = (0\#1) + (10\#0)$.

LO5. Let $\text{len}(x)$ be defined as the length of natural number x when written in binary. For example, $\text{len}(13) = 4$ since $(13)_2 = 1101$. Provide a *recursive* definition for $\text{len}(x)$. You may use any functions defined in any of the examples or exercises of the Models of Computation lecture. Hint: you may find it helpful to use $\underset{z\le y}{\exists}$, the bounded existential function, whose single input should be a predicate function $M(\vec{x}, y)$.

LO6. Do the following.

(a) Compute the Gödel number for program $P = J(1, 1, 2), T(2, 4), Z(3), S(6)$. Write your answer as a sum of powers of two minus 1 (see part b).

(b) Provide the URM program P whose Gödel number equals

$$2^{10} + 2^{23} + 2^{29} + 2^{77} - 1.$$