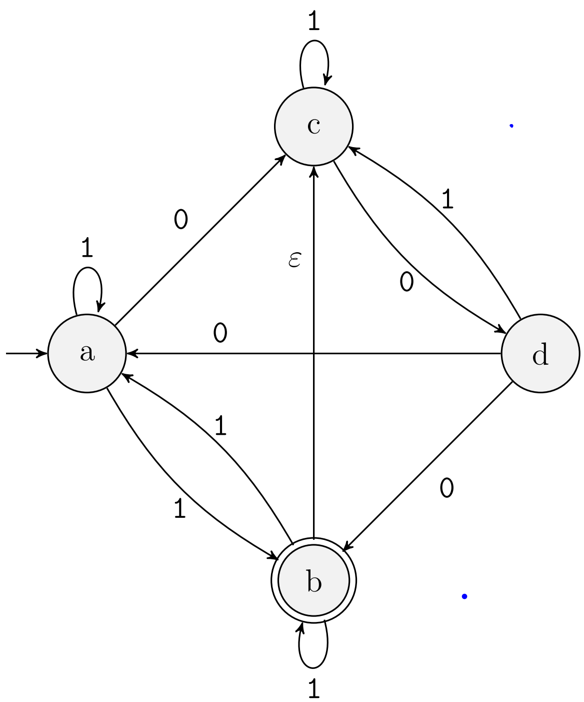
CECS 329, Learning Outcome Assessment 6, March 9th, Spring 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit solutions to at most 2 LO problems on separate sheets of paper.

Problems

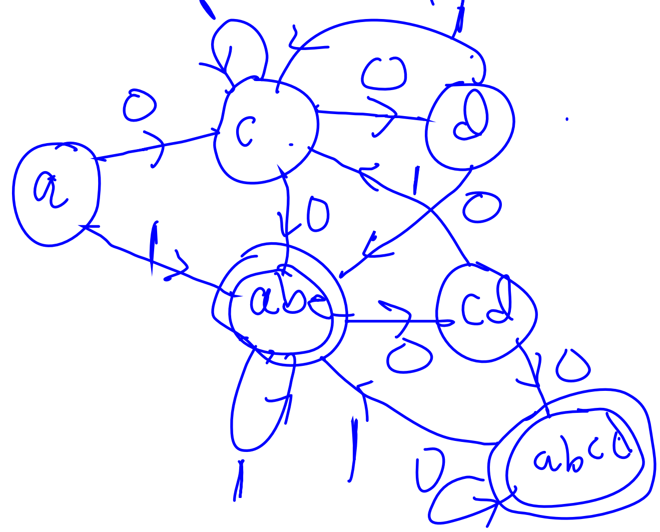
LO2. Do the following for the NFA N whose state diagram is shown below.



a) Solution

Q/Σ	0	1
a	c	abc
b	∅	abc
c	d	c
d	abc	c

b)



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

(c) Show the computation of M on input $w = 11001$.

a →1→ abc →1→ abc →0→ cd →0→ abcd →1→ abc ⇒ accept

LO3. Provide a regular expression that represents the set of words of the form

$$(w_1\#w_2) + (w_3\#w_4) + \cdots + (w_{2n-1}\#w_{2n}),$$

where each w_i, $i = 1, \ldots, 2n$, is a nonempty binary word. For example $(010\#1) + (11\#110)$ is one such word. Hint: the underlying alphabet is $\Sigma = \{0, 1, (,), \#, +\}$.

Solution.

$$(\{0,1\}^+\#\{0,1\}^+)[+(\{0,1\}^+\#\{0,1\}^+)]^*.$$

LO4. Do the following.

(a) Provide a context free grammar $G = (V, \Sigma, R, S)$ for which $L(G)$ is the set of words of the form

$$(w_1\#w_2) + (w_3\#w_4) + \cdots + (w_{2n-1}\#w_{2n}),$$

where each w_i, $i = 1, \ldots, 2n$, is a nonempty binary word. For example $w = (0\#1)+(10\#0)$ is one such word. Hint 1: the terminal set is $\Sigma = \{0, 1, (,), \#, +\}$. Hint 2: use the start symbol to generate the first left parenthesis, but use a different variable to generate all subsequent left parentheses.

Solution.

$$S \to (B\#B)X$$

$$X \to +(B\#B)X \mid \varepsilon.$$

$$B \to 0 \mid 1 \mid 0B \mid 1B$$

(b) Use G to provide a leftmost derivation of $w = (0\#1) + (10\#0)$.

Solution.

$$S \to (B\#B)X \to (0\#B)X \to (0\#1)X \to (0\#1) + (B\#B)X \to$$
$$(0\#1) + (1B\#B)X \to (0\#1) + (10\#B)X \to (0\#1) + (10\#0)X \to (0\#1) + (10\#0).$$

LO5. Let $\text{len}(x)$ be defined as the length of natural number x when written in binary. For example, $\text{len}(13) = 4$ since $(13)_2 = 1101$. Provide a *recursive* definition for $\text{len}(x)$. You may use any functions defined in any of the examples or exercises of the Models of Computation lecture. Hint: you may find it helpful to use $\underset{z \le y}{\exists}$, the bounded existential function, whose single input should be a predicate function $M(\vec{x}, y)$.

Solution.

Base case: $\text{len}(0) = 1$

Recursive case:

$$\text{len}(x+1) = \begin{cases} \text{len}(x) + 1 & \text{if } \underset{z \le x}{\exists}(x = 2^z - 1) \\ \text{len}(x) & \text{Otherwise} \end{cases}$$

LO6. Do the following.

(a) Compute the Gödel number for program $P = J(1,1,2), T(2,4), Z(3), S(6)$. Write your answer as a sum of powers of two minus 1 (see part b).

Solution.

$$\beta(J(1,1,2)) = 4\xi(0,0,1) + 3 = 4\pi(\pi(0,0),1) + 3 = 4\pi(0,1) + 3 = 4(2) + 3 = 11.$$

$$\beta(T(2,4)) = 4\pi(1,3) + 2 = 4(13) + 2 = 54.$$

$$\beta(Z(3)) = 4(2) = 8.$$

$$\beta(S(6)) = 4(5) + 1 = 21.$$

$$\gamma(P) = \tau(11, 54, 8, 21) = 2^{11} + 2^{66} + 2^{75} + 2^{97} - 1.$$

(b) Provide the URM program P whose Gödel number equals

$$2^{10} + 2^{23} + 2^{29} + 2^{77} - 1.$$

Solution.

$P = T(1,2), Z(4), S(2), J(1,2,2)$

For example, to get $J(1,2,2)$, we have $77 - 29 - 1 = 47$ and $47 \bmod 4 = 3$ which implies a Jump instruction. Also, $(47-3)/4 = 11$, and $\pi^{-1}(11) = (2,1)$ since $(11+1) = 2^2(2(1)+1)$. Finally, π^{-1} of the exponent 2 equals $(0,1)$ since $(2+1) = 2^0(2(1)+1)$. Therefore, we have $J(0+1, 1+1, 1+1) = J(1,2,2)$.