CECS 528, Exam 1, Pink Version, Fall 2023, Dr. Ebert

Rules for Completing the Problems

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION allowed when solving these problems. Make sure all these items are put away BEFORE looking at the problems. FAILURE TO ABIDE BY THESE RULES MAY RESULT IN A FINAL COURSE GRADE OF F.

Directions

Choose up to **six problems** to solve. Clearly mark each problem you want graded by placing an X or check mark in the appropriate box in the Grade(?) row of the table below. **If you don't mark any problems for us to grade or mark 7 or more problems, then we will record grades for the 6 that received the *fewest* points.**

Problem	1	2	3	4	5	6
Grade?						
Result						

Your Full Name:

Your Class ID:

1. Solve each of the following problems. Note: correctly solving these problems counts for passing LO1.

   a. Compute the multiplicative inverse of 25 mod 36. (10 pts)

   b. For the Strassen-Solovay primality test is $a = 11$ an accomplice or witness to the fact that $n = 15$ is not prime? Show all work. (15 pts)

2. Solve each of the following problems. Note: correctly solving these problems counts for passing LO2.

   a. Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 56T(n/4) + n^3$. (10 pts)

   b. Use the substitution method to prove that, if $T(n)$ satisfies

   $$T(n) = 4T(n/2) + n^2 \log n,$$

   Then $T(n) = \Omega(n^2 \log^2 n)$. (15 pts)

3. Solve each of the following problems. Note: correctly solving these problems counts for passing LO3.

   a. Recall the combine step of the `Minimum Distance Pair (MDP)` algorithm where, for each point P in the δ-strip, there is a $2\delta \times \delta$ rectangle whose bottom side contains P and is bisected by the vertical line that divides the points into left and right subsets. Explain why there can be at most 7 other points (from the problem instance) in this rectangle. (12 pts)

   b. Recall that the `Minimum Positive Subsequence Sum (MPSS)` problem admits a divide-and-conquer algorithm that, on input integer array a, requires computing the mpss of any subarray of a that contains both $a[n/2-1]$ and $a[n/2]$ (the end of a_{left} and the beginning of a_{right}). For

   $$a = 48, -37, 29, -33, 51, -64, 46, -34, 45, -36$$

   Provide the two sorted arrays a and b from which the minimum positive sum $a[i] + b[j]$ represents the desired mpss, for some i in the index range of a and some j within the index range of b. Also, demonstrate how the minimum positive sum $a[i] + b[j]$ may be computed in $\mathrm{O}(n)$ steps. (13 pts)

4. In this problem we assume that multiplication of an m-bit number with an n-bit number results in a product having $m+n$ bits, and that requires $O(mn)$ steps to compute. Using these assumptions, determine the worst-case running time for the following code that computes x^y, where we assume x is an m-bit number and y is an n-bit number. (25 pts)

```
prod = x;

for(i=1; i < y; i++)
    prod = prod*x;

return prod;
```

5. Recall the Master Equation

$$T(n) = \Theta(n^{\log_b a}) + \sum_{j=0}^{\log_b n - 1} a^j f(n/b^j).$$

Assuming n is a power of b, suppose that $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and that, for all $n \geq 1$, $af(n/b) \leq cf(n)$ for some constant positive $c < 1$, then prove that $T(n) = \Theta(f(n))$. (25 pts)

6. The Hadamard matrices H_0, H_1, $H_2 \ldots$ are recursively defined as follows. H_0 is the 1×1 matrix [1], and, for $k \geq 1$, H_k is the $2^k \times 2^k$ matrix

$$H_k = \left(\begin{array}{c|c} H_{k-1} & H_{k-1} \\ \hline H_{k-1} & -H_{k-1} \end{array} \right).$$

Describe an algorithm that computes the matrix-vector product $H_k v$ using $O(n \log n)$ operations, where v is a column vector of length $n = 2^k$. Assume that all the numbers involved are small enough so that basic arithmetic operations like addition and multiplication take unit time. Note: credit will not be awarded to descriptions that are ambiguous, make incorrect assumptions/conclusions, and/or do not achieve the desired bound on operations. (25 points)