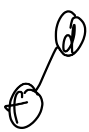
[05] a] $f_i \geq f_{i-1}$ since the algorithm always adds as much of an item as possible. Thus the fraction sequence has the form $1, \ldots, 1, f, 0, \ldots, 0$, where $f \in [0,1]$. In other words all of an item will be added so long as there is enough remaining capacity. This is followed by at most one item for which only a fraction f of the item can be added meaning the knapsack will result in being filled. Therefore all subsequent fraction must equal 0.

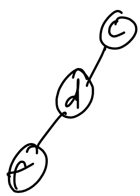
b] $f_k' > f_k$ means that the algorithm does not add as much of an item x_k as it could have, a contradiction since the algorithm always adds as much of an item as is physically possible.

c] since items until x_k are accounted for, next profitable item is x_{k+1} but adding $(f_k - f_k')w_k$ of this item yields a profit equal to $d_{k+1}(f_k - f_k')w_k \leq d_k(f_k - f_k')w_k$. Since the profit density d_{k+1} does not exceed that of d_k. Therefore, by replacing $(f_k - f_k')w_k$ units of the next most profitable item with $(f_k - f_k')w_k$ of x_k, the resulting knapsack is optimal.

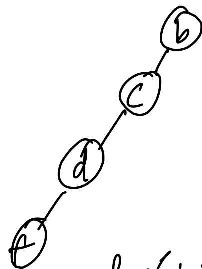
[06] step 1:



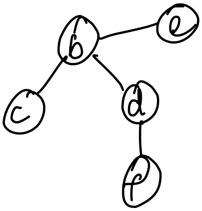
step 2: find f = d
find c = c



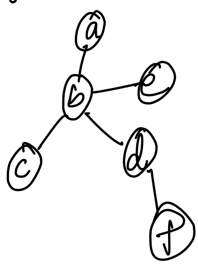
step 3: find b = b
find d = c



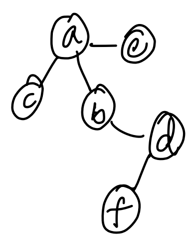
step 4: find d = b
find e = e



step 5:

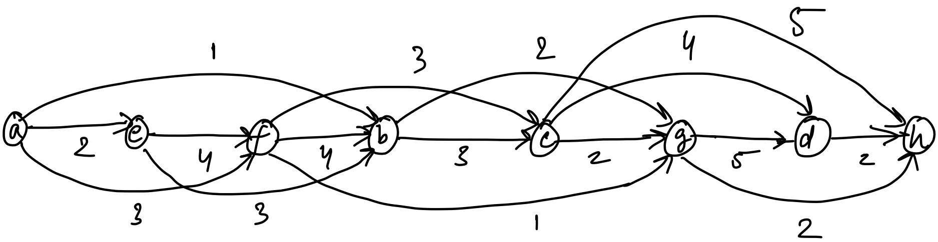


step 6: find c = a
find e = a



L07] a] $mc(u,v) = \begin{cases} \infty & \text{if } deg^+(v) = 0 \\ 0 & \text{if } u = v \\ \max\limits_{w \in E^+(v)} (c(w,v) + mc(u,w)) \end{cases}$

b]



c] $mc(a,a) = 0$

$mc(a,e) = \max[2 + mc(a,a)] = 2$

$mc(a,f) = \max[4 + mc(a,e), 3 + mc(a,a)] = 6$

$mc(a,b) = \max[4 + mc(a,f), 1 + mc(a,a), 3 + mc(a,e)] = 10$

$mc(a,c) = \max[3 + mc(a,b), 3 + mc(a,f)] = 13$

$mc(a,g) = \max[2 + mc(a,b), 2 + mc(a,c), 1 + mc(a,f)] = 15$

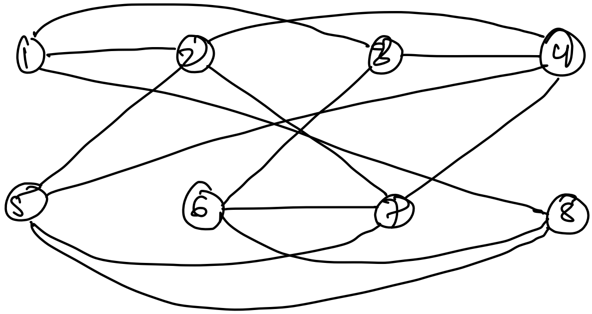
$mc(a,d) = \max[5 + mc(a,g), 4 + mc(a,c)] = 20$

$mc(a,h) = \max[2 + mc(a,d), 5 + mc(a,c), 2 + mc(a,g)] = \underline{22}$

L08] a] Refer pink solution.

L09] a] Problem a is mapping reducible to problem b, written $a \le_m b$ if exists a computable function $f: A \to B$ for which the solution to problem instance x of A is equal to the solution to problem instance $f(x)$ of B.

[Q9] b]



c] Independent set for G = {5, 2, 4, 7} = 4

Even for $\bar{G}$, max clique will be {5, 2, 4, 7}

∴ solution for $\bar{G}$ & G is same.