CECS 528, Learning Outcome Assessment 7, Pink, Fall 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit each solution on a separate sheet of paper.

Problems

LO3. Solve the following problems.

(a) The number of computational steps $T(n)$ required by Strassen's algorithm for multiplying two $n \times n$ matrix inputs satisfies the recurrence $T(n) = 7T(n/2) + n^2$. What accounts for the $O(n^2)$ number of steps in the dividing/combining portions of the algorithm? Explain.

(b) Draw the recursion tree that results when applying Mergesort to the array

$$a = 5, -2, 0, 7, 3, 11, 2, 9, 5, 6,$$

Label each node with the sub-problem to be solved at that point of the recursion. Assume arrays of size 1 and 2 are base cases. Assume that odd-sized arrays are split so that the left subproblem has one more integer than the right. Next to each node, write the solution to its associated subproblem.

LO4. Answer the following.

(a) The FFT algorithm owes its existence to what two properties that are possessed by the nth roots of unity when n is even?

(b) Compute $\text{DFT}_4(5, -4, 3, -2)$ using the FFT algorithm. Show the solution to each of the seven subproblem instances and, for each one, clearly represent it using DFT notation and apply the formula for computing it. Show all work.

LO5. Answer the following with regards to a correctness-proof outline for Dijkstra's algorithm.

(a) In relation to Dijkstra's algorithm, provide a definition for what it means to be i) an i-neighboring path from source s to an external vertex v, and ii) the i-neighboring distance $d_i(s, v)$ from source s to external vertex v. Hint: at this point in the algorithm i nodes have been added to the DDT.

(b) Using the definitions from part a, describe the greedy choice that is made in each round of Dijkstra's algorithm.

(c) If vertex v is chosen by Dijkstra in Round $i+1$, use part b to prove that $d(s, v) = d_i(s, v)$. Hint: if i-neighboring path P from s to v has cost $d_i(s, v)$ and Q is any other path from s to v, explain why $\text{cost}(Q) \geq d_i(s, v)$.

LO6. Recall the use of the disjoint-set data structure for the purpose of improving the running time of the `Unit Task Scheduling` algorithm. For the set of tasks

Task	a	b	c	d	e	f
Deadline Index	2	3	0	3	4	3
Profit	60	50	40	30	20	10

For each task, show the M-Tree forest after it has been inserted (or at least has attempted to be inserted in case the scheduling array is full). Note that the earliest possible deadline index is 0, meaning that the earliest slot in the schedule array has index 0. Also, assume that an insert attempt that takes place at index i results in the function call `root`(i). Finally, to receive credit, your solution should show six different snapshots of the M-Tree forest.

LO7. Answer/Solve the following questions/problems.

(a) The dynamic-programming algorithm that solves the `Matrix-Chain Multiplication` optimization problem defines a recurrence for the function $\mathrm{mc}(i, j)$. In words, what does $\mathrm{mc}(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $\mathrm{mc}(i, j)$.

(c) Apply the recurrence from Part b to the dimension sequence 3,4,6,1,5. Show the matrix of subproblem solutions and use it to provide an optimal parenthesization. For each subproblem solution, make sure to indicate the value of k that produced the solution.