L01] a] $4^{1536} + 9^{4824}$

$4^{1536} \bmod 7$

$\because 4 = 2^2 \Rightarrow 4^{1536} = 2^{3072}$

$2^3 \equiv 1 \bmod 7$

$3072/3 = 1024$

$4^{1536} \equiv 1 \bmod 7$

$9^{4829} \bmod 7$

$\because 9 = 3^3 \Rightarrow 9^{4824} = 3^{14472}$

$3^3 = 1 \bmod 7$

$\frac{14472}{3} = 4824$

$9^{4829} \equiv 1 \bmod 7$

$\therefore 4^{1536} + 9^{4824} \equiv \underline{\underline{2}} \bmod 7.$

b]

$a = 3 \quad n = 5$

$$a^{\frac{n-1}{2}} \equiv \left(\frac{a}{n}\right) \bmod n$$

$$\Rightarrow 3^2 = \frac{3}{5} \bmod 5$$

$$\Rightarrow 9 \bmod 5 = 4$$

$$\Rightarrow \frac{3}{5} \bmod 5 = \frac{5}{3} \bmod 5$$

$$= \frac{3(1)+2}{3} \bmod 5$$

$$= \frac{2}{3} \bmod 5$$

$= -1 \bmod 5$

$\therefore \quad 4 \equiv -1 \bmod 5$

$\therefore$ LHS = ~~R~~HS

a is an accomplice for 5 being prime.

2Q2] a) $n^{\log_3 12} > n^{\log_{12} 3}$

case 1 $\quad T(n) = \Theta(n^{\log_3 12})$

b] inductive assumption $\Rightarrow T(n) \geq ck\log^2 k$

$\therefore \cancel{2}c\frac{n}{\cancel{2}}\log^2\frac{n}{2} + 6n\log n \geq cn\log^2 n$

$\Rightarrow cn(\log n - \log 2)^2 + 6n\log n \geq cn\log^2 n$

$\Rightarrow cn(\log^2 n - 2\log n + 1) + 6n\log n \geq cn\log^2 n$

$\Rightarrow \cancel{cn\log^2 n} - 2cn\log n + cn + 6n\log n \geq \cancel{cn\log^2 n}$

$\Rightarrow n(-2c\log n + c + 6\log n) \geq 0$

$6\log n \geq 2c\log n - c$

$$\frac{6\log n}{2\log n - 1} \geq c$$

if $c=3$

$$6\log n \geq 3(2\log n - 1)$$

$$6\log n \geq 6\log n - 3$$

$$0 \geq -3$$

$\therefore$ with $c=3$ the equation holds true for n sufficiently large.

[03] a] Since there are 25 members of a less than median we can say that in eq.

$$E[X \mid Y = 26] = T(26-1) + T(n-26) + O(n)$$

Since i is the median index, we can say that $i = 26$.

b] a = 7, 10, 1, 2, 8, 9, 11, 4, 3, 5, 6.

median(a[0], a[10], a[5]) = 7.

7	10	1	2	8	9	11	4	3	5	6
6	10	1	2	8	9	11	4	3	5	7
6	5	1	2	8	9	11	4	3	10	7
6	5	1	2	3	9	11	4	8	10	7
6	5	1	2	3	4	11	9	8	10	7
6	5	1	2	3	4	7	9	8	10	11

$$\underbrace{6\ 5\ 1\ 2\ 3\ 4}_{\text{Left}}\ \underbrace{7}_{\text{M}}\ \underbrace{9\ 8\ 10\ 11}_{\text{right}}$$

L04] a] $DFT_8(7,-10,1,2,-8,9,-11,4)$

$\Rightarrow DFT_4(7,1,-8,-11)$ odd $\qquad DFT_4(-10,2,9,4)$ even

$\Rightarrow DFT_2(7,-8)$ odd $\quad DFT_2(1,-1)$ even $\quad DFT_2(-10,9)$ odd $\quad DFT_2(2,4)$ even

$\Rightarrow DFT_1(7) \quad DFT_1(1) \quad DFT_1(-10) \quad DFT_1(2)$

$DFT_1(-8) \quad DFT_1(-1) \quad DFT_1(9) \quad DFT_1(4)$

b] $DFT^{-1}(2,1,-3,4)$

$DFT_1^{-1}(2)=2,\ DFT_1^{-1}(1)=1,\ DFT_1^{-1}(-3)=3,\ DFT_1^{-1}(4)=4$

$DFT_2^{-1}(2,-3)=2-3x \quad \therefore y_0=\frac{1}{2}[(2,2)\odot(-3,-3)(1,-1)]=\frac{1}{2}[-1,5,-1,5]$

$DFT_2^{-1}(1,4)=1+4x=y_1=\frac{1}{2}[(1,1)\odot(4,4)(1,-1)]=\frac{1}{2}[5,-3,5,-3]$

$\frac{1}{2}[y_0\odot\vec{w_0}\,y_1]=\frac{1}{2}\left(\left[\frac{-1}{2},\frac{5}{2},\frac{-1}{2},\frac{5}{2}\right]\odot\left[\frac{5}{2},\frac{-3}{2},\frac{5}{2},\frac{-3}{2}\right][1,-i,-1,i]\right)$

$=\frac{1}{2}\left(\frac{4}{2},\frac{5+3i}{2},\frac{-6}{2},\frac{5-3i}{2}\right)$

$=\frac{2}{2},\frac{5+3i}{4},\frac{-3}{2},\frac{5-3i}{4}$

$=\left(1,\frac{5+3i}{4},-\frac{3}{2},\frac{5-3i}{4}\right)$

L05] a] If $t\_k < t <$ deadline of a_k then since the algorithm schedules a_k at t_k, the algorithm passed on t since there was already a task a_i scheduled at t, where i is a member of the set $\{1, 2, \ldots, k-1\}$. But S-opt agrees with S upto the scheduling of tasks $1, 2, \ldots, k-1$ & so S opt also schedules a_i at t, a contradiction.

b] Given that task a scheduled at t_k is getting completed before its deadline, swapping it to a time 't' such that $t < t_k$ will have no effect as task a_k will still get finished prior to deadline

c] Assume (a, t) is in S opt but not S. Then $t <$ deadline a & since S subset of S opt, S never schedules a task at time t. But the UTS algorithm attempts to schedule all tasks including task a, & thus should have scheduled all tasks scheduled a within the interval since t was available. Therefore S opt cannot have more scheduled tasks than S.