CECS 528, Learning Outcome Assessment 4, Pink, Fall 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit each solution on a separate sheet of paper.

Problem

LO1. Solve the following problems.

(a) Evaluate $2^{175} \bmod 127$.

(b) For the Strassen-Solovay primality test, is $a = 2$ an accomplice or witness to the fact that $n = 5$ is not prime? Show all work.

LO2. Solve the following problems.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 8T(n/3) + n^{\log_3 7}$. Defend your answer.

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = 4T(n/2) + n^2 \log n,$$

Then $T(n) = \mathrm{O}(n^2 \log^2 n)$.

LO3. Solve each of the following problems.

(a) Recall the combine step of the `Minimum Distance Pair (MDP)` algorithm where, for each point P in the δ-strip, there is a $2\delta \times \delta$ rectangle whose bottom side contains P and is bisected by the vertical line that divides the points into left and right subsets. Explain why there can be at most 7 other points (from the problem instance) in this rectangle.

(b) For the (non-randomized) `Find-Statistic` algorithm, determine the value of the pivot M (at the top level of recursion) for

$$a = 56, 29, 95, 46, 23, 18, 78, 58, 17, 99, 44, 74, 59, 37, 26, 83, 66, 45, 19, 51, 66, 92, 34$$

and $k = 7$. Show work.

LO4. Solve each of the following problems.

(a) Given degree-3 polynomial $p(x)$ where $p(1) = 7$, $p(i) = 4 + 2i$, $p(-1) = -4$, and $p(-i) = 4 - 2i$. How are the coefficients of p obtained via a Fourier transform? Explain. Note: you do *not* need to evaluate the transform.

(b) Compute $\mathrm{DFT}_4(-2, 4, 0, 5)$ using the FFT method. Show the solution to each of the subproblem instances (including the original problem instance) that must be solved.