CECS 528, Learning Outcome Assessment 11, Yellow, Fall 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit each solution on a separate sheet of paper.

Problems

LO7. Solve the following problems.

(a) The dynamic-programming algorithm that solves the `Longest Common Subsequence (LCS)` optimization problem defines a recurrence for the function $\text{lcs}(i, j)$. In words, what does $\text{lcs}(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $\text{lcs}(i, j)$.

(c) Apply the recurrence from Part b to the words $u =$ baabab and $v =$ bbbaaa. Show the matrix of subproblem solutions and use it to provide an optimal solution.

LO8. Do/answer the following.

(a) Draw the implication graph $G_{\mathcal{C}}$ associated with the `2SAT` instance

$$\mathcal{C} = \{(x_1, x_3), (\overline{x}_1, \overline{x}_4), (x_2, x_5), (\overline{x}_2, \overline{x}_5), (\overline{x}_3, x_4), (\overline{x}_3, \overline{x}_4), (x_3, x_5)\}.$$

(b) Apply the improved `2SAT` algorithm to obtain a satisfying assignment for $\mathcal{C}$. When deciding on the next reachability set R_l to compute, follow the literal order $l = x_1, \overline{x}_1, \ldots, x_5, \overline{x}_5$. For each consistent reachability set encountered, provide the partial assignment α_{R_l} associated with R_l and draw the reduced implication graph before continuing to the next reachability set. Note: do *not* compute the reachability set for a literal that has already been assigned a truth value. Provide a final assignment α and verify that it satisfies all the clauses.

(c) Using the original `2SAT` algorithm, suppose $\texttt{reachable}(G_{\mathcal{C}}, x_2, \overline{x}_2)$ evaluates to 1. Then what must be true about any assignment α that satisfies $\mathcal{C}$? Explain.

LO9. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B. Hint: do *not* assume A and B are decision problems.

(b) For the mapping reduction f : `Maximum Bipartite Matching` $\to$ `Maximum Flow`, draw $f(G)$ for `MBM` instance $G = (U, V, E)$, where $U = \{u_1, u_2, u_3, u_4\}$, $V = \{v_1, v_2, v_3, v_4\}$, and

$$E = \{(u_1, v_1), (u_1, v_2), (u_1, v_4), (u_2, v_1), (u_2, v_3),$$

$$(u_2, v_4), (u_3, v_1), (u_3, v_3), (u_4, v_1), (u_4, v_3)\}.$$

(c) Verify that both G and $f(G)$ have the same solution, where we assume that a "solution" to each problem instance is a nonnegative integer. Defend your answer by providing details of each solution.

LO10. An instance of `Set Cover (SC)` is a triple $(\mathcal{S}, m, k)$, where $\mathcal{S} = \{S_1, \ldots, S_n\}$ is a collection of n subsets, where $S_i \subseteq \{1, \ldots, m\}$, for each $i = 1, \ldots, n$, and a nonnegative integer k. The problem is to decide if there are k subsets $S_{i_1}, \ldots, S_{i_k}$ for which

$$S_{i_1} \cup \cdots \cup S_{i_k} = \{1, \ldots, m\}.$$

(a) For a given instance $(\mathcal{S}, m, k)$ of `SC` describe a certificate in relation to $(\mathcal{S}, m, k)$.

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance $(\mathcal{S}, m, k)$, and ii) a certificate for $(\mathcal{S}, m, k)$ as defined in part a, and decides if the certificate is valid for $(\mathcal{S}, m, k)$.

(c) Provide appropriate size parameters for `SC`. Hint: there are two of them.

(d) Use the size parameters from part c to describe the running time of your verifier from part b. Defend your answer.

LO11. Recall the mapping reduction from `SAT` to `3SAT` described in lecture.

(a) Given the `SAT` instance $F(x_1, x_2, x_3) = \overline{x}_1 \vee (x_2 \wedge \overline{x}_3)$, draw its parse tree and provide the associated Boolean formula G that is satisfiability equivalent to F and serves as the beginning step of the reduction. Hint: formula G introduces y-variables.

(b) Rewrite formula G by making use of the logical identity

$$(P \leftrightarrow Q) \Leftrightarrow [(P \rightarrow Q) \wedge (Q \rightarrow P)]$$

(c) Rewrite the formula from part b by making use of the logical identity

$$(P \rightarrow Q) \Leftrightarrow (\overline{P} \vee Q).$$

(d) Rewrite the formula from part c by performing one or more applications of De Morgan's rule.

(e) Rewrite the formula from part d by performing one or more applications of the distributive rule in order to obtain an AND of OR's. Then convert the AND of OR's to an AND of ternary (i.e. three) OR's and use `3SAT` notation to complete the reduction.