CECS 528, Learning Outcome Assessment 11, Pink, Fall 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit each solution on a separate sheet of paper.

Problems

LO7. Answer the following.

(a) Provide the dynamic-programming recurrence for computing the distance $D(u, v)$, from a vertex u to a vertex v in a directed acyclic graph (DAG) $G = (V, E, c)$, where $c(x, y)$ gives the cost of edge $e = (x, y)$, for each $e \in E$. Hint: step *backward* from v, rather than forward from u.

(b) Draw the vertices of the following DAG G in a linear left-to-right manner so that the vertices are topologically sorted, meaning, if (u, v) is an edge of G, then u appears to the left of v. The vertices of G are a-h, while the weighted edges of G are

$$(a, b, 2), (a, e, 5), (a, f, 5), (b, c, 5), (b, g, 2), (c, d, 1), (c, g, 4), (c, h, 5), (d, h, 5), (e, b, 1), (e, f, 3),$$

$$(f, b, 5), (f, c, 2), (f, g, 1), (g, d, 2), (g, h, 3).$$

(c) Starting from left to right in topological order, use the recurrence to compute

$$d(a, a), \ldots, d(a, h).$$

LO8. Do/answer the following.

(a) Draw the implication graph $G_{\mathcal{C}}$ associated with the `2SAT` instance

$$\mathcal{C} = \{(\overline{x}_1, x_2), (\overline{x}_1, \overline{x}_3), (x_1, x_4), (x_2, x_4), (\overline{x}_2, \overline{x}_4), (\overline{x}_2, \overline{x}_5), (\overline{x}_3, \overline{x}_4)\}.$$

(b) Apply the improved `2SAT` algorithm to obtain a satisfying assignment for $\mathcal{C}$. When deciding on the next reachability set R_l to compute, follow the literal order $l = x_1, \overline{x}_1, \ldots, x_5, \overline{x}_5$. For each consistent reachability set encountered, provide the partial assignment α_{R_l} associated with R_l and draw the reduced implication graph before continuing to the next reachability set. Note: do *not* compute the reachability set for a literal that has already been assigned a truth value. Provide a final assignment α and verify that it satisfies all the clauses.

(c) Suppose `2SAT` instance $\mathcal{C}$ is satisfiable and uses 336 variables and 615 clauses. Using the original `2SAT` algorithm, what is the *least* number of queries to a `Reachability` oracle that needs to be made in order to establish $\mathcal{C}$'s satisfiability. In other words, if we make fewer than this number of queries then it is possible that the `2SAT` instance $\mathcal{C}$ may be unsatisfiable. Explain.

LO9. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Subset Sum` $\rightarrow$ `Set Partition`, determine $f(S, t)$ for `Subset Sum` instance $(S = \{12, 15, 17, 24, 26, 27\}, t = 70)$. Show work.

(c) Verify that both (S, t) and $f(S, t)$ are either both positive instances or both negative instances of their respective decision problems. If both are positive, then provide valid certificates for each. Otherwise, explain why neither has a valid certificate.

LO10. An instance of the `Quadratic Residue (QR)` decision problem is a triple (a, c, m) of positive integers, where $a, c \leq m$, and the problem is to decide if there is an $1 \leq x \leq c$ for which $x^2 \equiv a \bmod m$.

(a) For a given instance (a, c, m) of `QR` describe a certificate in relation to (a, c, m).

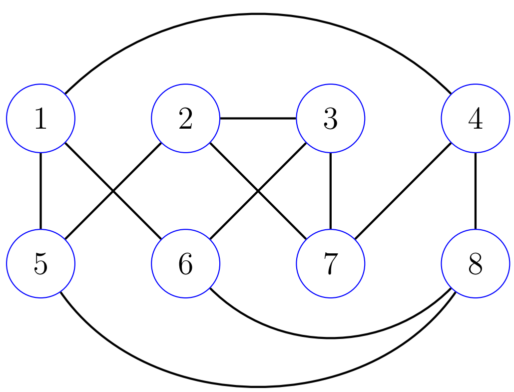
(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (a, c, m), and ii) a certificate for (a, c, m) as defined in part a, and decides if the certificate is valid for (a, c, m).

(c) Suppose m is a b-bit number, explain why b is a more appropriate size parameter than m.

(d) Use the b size parameter to describe the running time of your verifier from part b. Hint: make reference to the complexity of certain arithmetic algorithms.

LO11. Recall the mapping reduction f : `HC` $\rightarrow$ `TSP` from `Hamilton Cycle` to `Traveling Salesperson` described in lecture.

(a) Given the `HC` instance G shown below, draw $f(G)$ and indicate its k value.



(b) By providing valid certificates for each, verify that both G and $f(G)$ are positive instances of their respective decision problems. Show work and explain.