CECS 528, Learning Outcome Assessment 10, Pink, Fall 2023, Dr. Ebert

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit each solution on a separate sheet of paper.

Problems

LO6. Recall the use of the disjoint-set data structure for the purpose of improving the running time of the `Unit Task Scheduling` algorithm. For the set of tasks

Task	a	b	c	d	e	f
Deadline Index	2	1	2	4	3	4
Profit	60	50	40	30	20	10

For each task, show the M-Tree forest after it has been inserted (or at least has attempted to be inserted in case the scheduling array is full). Note that the earliest possible deadline index is 0, meaning that the earliest slot in the schedule array has index 0. Also, assume that an insert attempt that takes place at index i results in the function call `root`(i). Finally, to receive credit, your solution should show six different snapshots of the M-Tree forest.

LO7. Answer the following.

(a) The dynamic-programming algorithm that solves the `0-1 Knapsack` optimization problem defines a recurrence for the function $p(i,c)$. In words, what does $p(i,c)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $p(i,c)$.

(c) Apply the recurrence from Part b to a knapsack having capacity $M = 10$ and items

item	**weight**	**profit**
1	5	30
2	4	30
3	1	20
4	4	40
5	5	30
6	5	60

Show the matrix of subproblem solutions and use it to provide an optimal set of items.

LO8. Do/answer the following.

(a) Draw the implication graph $G_{\mathcal{C}}$ associated with the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_5), (x_1, x_5), (\overline{x}_2, \overline{x}_3), (\overline{x}_2, \overline{x}_4), (x_2, x_4), (x_2, \overline{x}_4), (\overline{x}_3, x_4), (x_3, x_5)\}.$$

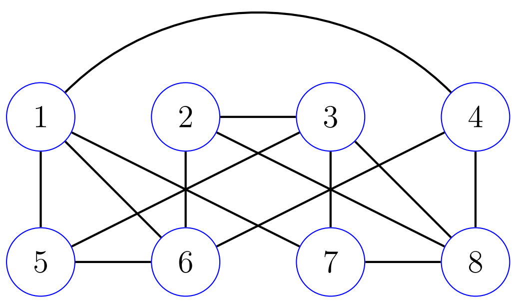
(b) Apply the improved `2SAT` algorithm to obtain a satisfying assignment for $\mathcal{C}$. When deciding on the next reachability set R_l to compute, follow the literal order $l = x_1, \overline{x}_1, \ldots, x_5, \overline{x}_5$. For each consistent reachability set encountered, provide the partial assignment α_{R_l} associated with R_l and draw the reduced implication graph before continuing to the next reachability set. Note: do *not* compute the reachability set for a literal that has already been assigned a truth value. Provide a final assignment α and verify that it satisfies all the clauses.

(c) Suppose `2SAT` instance $\mathcal{C}$ is unsatisfiable and has 234 variables and 754 clauses. Using the original `2SAT` algorithm, what is the *least* number of queries to a `Reachability` oracle that needs to be made in order to establish $\mathcal{C}$'s unsatisfiability. In other words, it is possible that $\mathcal{C}$ could be proved unsatisfiable after this number of queries.

LO9. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction fromm problem A to problem B.

(b) Suppose $(G, k = 3)$ is an instance of the `Vertex Cover` decision problem, where G is drawn below. Draw $f(G, k)$, where f is the mapping reduction from `Vertex Cover` to the `Half Vertex Cover` decision problem.



(c) Verify that f is valid for input (G, k) in the sense that both (G, k) and $f(G)$ are either both positive instances or both negative instances of their respective problems. Defend your answer. Hint: it takes two vertices to cover each edge of a triangle in a graph.

LO10. An instance of the `Boolean Vector Sum (BVS)` decision problem is a pair (B, k) where B is a set of n Boolean vectors, each having length equal to $m > 0$, and k is a nonnegative integer. The problem is to decide if B has a subset of k vectors $\{v_1, \ldots, v_k\}$ for which

$$v_1 \vee v_2 \vee \cdots \vee v_k = (\underbrace{1, \ldots, 1}_{m \text{ times}}),$$

where operation $\vee$ represents bitwise OR.

(a) For a given instance (B, k) of `BVS` describe a certificate in relation to (B, k).

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (B, k), ii) a certificate for (B, k) as defined in part a, and decides if the certificate is valid for (B, k).

(c) Provide size parameters that may be used to measure the size of an instance of `BVS`.

(d) Use the size parameters from part c to describe the running time of your verifier from part b. Defend your answer in relation to the algorithm you provided for the verifier.