CECS 329, Exam 1, September 21st, Fall 2023, Dr. Ebert

Rules for Completing the Problems

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION allowed when solving these problems. Make sure all these items are put away BEFORE looking at the problems. FAILURE TO ABIDE BY THESE RULES MAY RESULT IN A FINAL COURSE GRADE OF F.

Directions

Choose up to **six problems** to solve. Clearly mark each problem you want graded by placing an 'X' or check mark in the appropriate box. **If you don't mark any problems for us to grade or mark 7 or more problems, then we will record grades for the 6 that received the *fewest* points.**

Problem	1	2	3	4	5	6
Grade?						
Result						

Your Full Name:

Your Class ID:

1. Answer the following. Note: correctly solving this problem counts for passing LO1.

   a. Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B. (5 pts)

   b. For the mapping reduction f : `Subset Sum` $\rightarrow$ `Set Partition`, based on the reduction provided in lecture, determine $f(S, t)$ for `Subset Sum` instance

   $$(S = \{4, 7, 15, 19, 22, 38, 44, 45\}, t = 111).$$

   Show work. (10 pts)

   c. Verify that $A = \{22, 44, 45\}$ is a valid certificate for (S, t), and use A to construct a valid certificate for $f(S, t)$. Explain and show work. (10 pts)

2. An instance of `Set Cover` is a triple $(\mathcal{S}, m, k)$, where $\mathcal{S} = \{S_1, \ldots, S_n\}$ is a collection of n subsets, where $S_i \subseteq \{1, 2, \ldots, m\}$, for each $i = 1, \ldots, n$, and k is a nonnegative integer. The problem is to decide if there are k subsets $S_{i_1}, \ldots, S_{i_k}$ for which

$$S_{i_1} \cup \cdots \cup S_{i_k} = \{1, 2, \ldots, m\}.$$

Note: correctly solving this problem counts for passing LO2.

a. Verify that $(\mathcal{S}, m, k)$ is a positive instance of `Set Cover`, where $m = 9$, $k = 4$, and

$$\mathcal{S} = \{\{1,3,5\}, \{3,7,9\}, \{2,4,5\}, \{2,6,7\}, \{6,7,9\}, \{2,7,9\}, \{1,3,7\}, \{4,5,8\}\}.$$

(5 pts)

b. For a given instance $(\mathcal{S}, m, k)$ of `Set Cover` describe a certificate in relation to $(\mathcal{S}, m, k)$. (5 pts)

c. Provide a semi-formal verifier algorithm that takes as input i) an instance $(\mathcal{S}, m, k)$, ii) a certificate for G as defined in part b, and decides if the certificate is valid. (5 pts)

d. Provide size parameters that may be used the measure the size of an instance of `Set Cover`. Hint: there should be two of them. (5 pts)

e. Use the size parameters from part d to describe the running time of your verifier from part c. Defend your answer in relation to the algorithm you provided for the verifier. (5 pts)

3. Answer the following. Note: correctly solving at least three parts counts for passing LO3.

a. Recall the mapping reduction $f(\mathcal{C}) = (S, t)$, where f maps an instance of `3SAT` to an instance of the `Subset Sum` decision problem. Given `3SAT` instance

$$\mathcal{C} = \{c_1 = (x_1, x_2, x_4), c_2 = (\overline{x}_2, x_3, \overline{x}_4), c_3 = (\overline{x}_1, x_2, x_3), c_4 = (\overline{x}_1, \overline{x}_3, \overline{x}_4),$$

What is the cardinality of S? What is the value of t? Explain and/or Show work. (5 pts)

b. Continuing from part a, is (S, t) a positive intance? If not, explain why. If yes, provide a subset of S that sums to target t. (6 pts)

c. Recall the mapping reduction $f(G) = (G', k)$, where f maps an instance of `Hamilton Cycle` to an instance of the `TSP` decision problem. If G has 8 vertices, then how many edges does G' have? What is the value of k? Explain. (7 pts)

d. Continuing from part c, if G has a Hamilton path, but no Hamilton cycle, then what is the total cost of the least-cost Hamilton cycle in G'. In other words, determine the cost of the most cost-efficient cycle that appears in G'. Explain. (7 pts)

4. Answer the following. Note: scoring 14 or more points counts for passing LO4.

a. Write either P, NP, or co-NP next to each of the following decision problems in terms of which one best characterizes the complexity of the problem. Note: there is only one best answer for each. Two points each.

i. An instance of `Triangle` is a simple graph G and the problem is to decide if G has a 3-clique.

ii. An instance of `Tired Traveling Salesperson (TTSP)` is a complete weighted graph G and an integer $k \geq 0$. The problem is to decide if every Hamilton Cycle in G has a cost that exceeds k.

iii. An instance of `Composite` is a natural number $n \geq 2$, and the problem is to decide if n is a composite number, i.e. is divisible by a number $2 \leq d \leq n-1$.

iv. An instance of `Quadratic Diophantine` is a triple of positive integers (a, b, c) and the problem is to decide if there are positive integers x and y for which $ax^2 + by = c$.

b. Circle the correct answer. Which of the following certificates is best suited for checking if some simple graph $G = (V, E)$ is a positive instance of `Hamilton Cycle`. (8 points)

i. Certificate C is a permutation of all n vertices of G.

ii. Certificate C is a subset of n edges of G, where $n = |V|$.

iii. Certificate C is a permutation of all m edges of G.

iv. Certificate C is a subset of n edges of G, where $n = |V|$.

c. Circle the correct answer. Which of the following is a true statement (9 points)

i. If $A \leq_m^p B$, $A \in \mathtt{P}$, and B is NP-complete, then $B \in \mathtt{P}$.

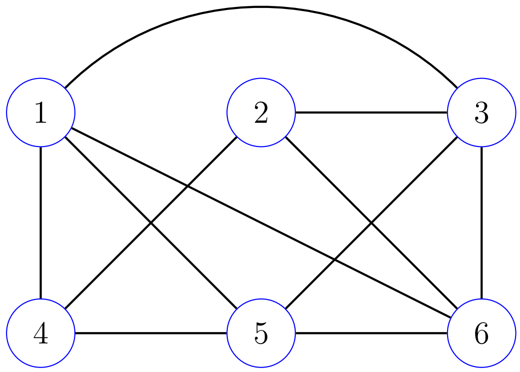
ii. If $A \leq_m^p B$, A is NP-complete, $B \in \mathtt{NP}$, then $\mathtt{P} \neq \mathtt{NP}$.

iii. If $A \leq_m^p B$, A is NP-complete, $B \in \mathtt{P}$, then $\mathtt{P} = \mathtt{NP}$.

iv. If $A \leq_m^p B$, $A \in \mathtt{P}$, and B is NP-complete, then $\mathtt{P} = \mathtt{NP}$.

5. In lecture we showed a polynomial-time mapping reduction $f : \texttt{HP} \to \texttt{HC}$.

   a. Draw the image of $f(G, 1, 6)$, where G is the graph shown below. Hint: you may represent your image by modifying G. (5 pts)



   b. Provide a complete description of a valid polynomial-time mapping reduction $g : \texttt{HC} \to \texttt{HP}$ from `Hamilton Cycle` to `Hamilton Path`. Function g must map a graph G to a triple (G', a, b), where a and b are vertices of G' and G has a Hamilton Cycle iff there is a Hamilton path from a to b. Use the graph from part a to demonstrate your reduction. Hint: clone a vertex of G. Argue that your reduction is valid. (20 pts)

6. The `Half Clique` decision problem is the problem of deciding if a simple graph $G = (V, E)$ has a Clique of size $|V|/2$.

   a. Describe an embedding reduction f from `Half Clique` to `Clique`. (10 points)

   b. Now describe a contraction reduction from `Clique` to `Half Clique` and demonstrate it for the `Clique` instance $(G, k = 4)$, where G is the graph shown below. (15 points)