CECS 329, Learning Outcome Assessment 9, November 9th, Fall 2023, Dr. Ebert

Problems

LO5. A natural number $x \geq 1$ is said to be `perfect` iff the sum of all its divisors (that are less than x) add to x. For example, 28 is perfect since

$$1 + 2 + 4 + 7 + 14 = 28.$$

Using any of the primitive recursive functions defined in the Models of Computation lecture, show that the predicate function $\texttt{perfect}(x)$ is primitive recursive, where $\texttt{perfect}(x) = 1$ iff x is perfect.

LO6. Solve the following problems.

(a) Compute the Gödel number for program $P = T(4,3), J(2,2,2), S(7), Z(4)$. Write your answer as a sum of powers of two minus 1 (see part b for an example). Show all work.

(b) Provide the URM program P whose Gödel number equals

$$2^9 + 2^{41} + 2^{50} + 2^{73} - 1.$$

Show all work.

LO7. Answer/solve the following.

(a) When simulating the computation $P_x(y)$, what is the only step in the simulation for which universal program P_U makes use of input y? Explain.

(b) A universal program P_U is simulating a program that has 77 instructions and whose Gödel number is

$$x = 2^{31} + 2^{32} + 2^{43} + 2^{83} + 2^{128} + 2^{216} + \cdots + 2^{c_{77}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^5 + 2^9 + 2^{11} + 2^{14} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO8. Answer the following.

(a) Define what it means to be a positive instance of decision problem `Self Accept`. Hint: "Gödel number x is a positive instance of `Self Accept` iff ______."

(b) The goal is to show that `Self Accept` is undecidable. We assume it is decidable by assuming that its characteristic function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a positive instance of Self Accept} \\ 0 & \text{if } x \text{ is a negative instance of Self Accept} \end{cases}$$

is total computable. Provide the definition for how to compute the "antagonist" function $g(x)$ based on the value of $f(x)$.

(c) By writing the values of $g(0), g(1), \ldots$ in the appropriate table cells (next to the number in that cell), verify that function g is different from each computable function ϕ_i, $i = 0, 1, \ldots$, Why does this create a contradiction?

index\input x	0	1	2	$\cdots$	i	$\cdots$	**self accepting?**
$\phi_0(x)$	$\uparrow$	12	7	$\cdots$	$\uparrow$	$\cdots$	no
$\phi_1(x)$	8	87	36	$\cdots$	96	$\cdots$	yes
$\phi_2(x)$	7	5	0	$\cdots$	$\uparrow$	$\cdots$	yes
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
$\phi_i(x)$	0	32	65	$\cdots$	$\uparrow$	$\cdots$	no
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$

LO9. An instance of the decision problem `Infinite Range` is a Gödel number x, and the problem is to decide if function ϕ_x has an infinite range, meaning that ϕ_x has an infinite number of distinct outputs. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } \phi_x \text{ has an infinite range} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where e_1 is the Gödel number of the program that computes the function $\phi_{e_1}(y) = \lfloor \sqrt{y} \rfloor$.

ii. $x = e_2$, where e_2 is the Gödel number of the program that computes the function $\phi_{e_2}(y) = e_2$.

iii. $x = e_3$, where e_3 is the Gödel number of the program that computes $g(x)$ (assuming that $g(x)$ is URM computable).

(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not ϕ_x has an infinite range. Do this by writing a program P that uses g and makes use of the `self` programming concept. Then show how P creates a contradiction.