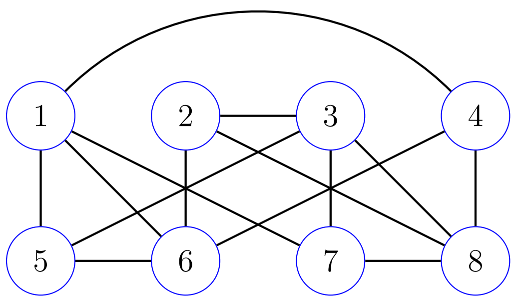
CECS 329, Learning Outcome Assessment 2, September 7th, Fall 2023, Dr. Ebert

Problems

LO1. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction fromm problem A to problem B. Hint: do *not* assume that A and B are decision problems.

(b) The simple graph $G = (V, E)$ shown below is an instance of the `Maximum Independent Set (MIS)` optimizationn problem. Draw $f(G)$, where f is the mapping reduction from `MIS` to `Maximum Clique` provided in lecture.



(c) Verify that f is valid for input G in the sense that both G and $f(G)$ have the same solution. Hint: for both problems please assume that a "solution" is represented by a subset of vertices.

LO2. An instance of the `Graph Matching` decision problem is a simple graph $G = (V, E)$, and the problem is to decide if G has a **matching**, i.e. a set $M \subseteq E$ of $|V|/2$ edges for which every vertex of G is incident with exactly one edge $e \in M$. For example, the graph G shown above is a positive instance of `Graph Matching`, since $M = \{(1, 7), (2, 8), (3, 5), (4, 6)\}$ is a matching for G.

(a) For a given instance $G = (V, E)$ of `Graph Matching` describe a certificate in relation to G.

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance $G = (V, E)$, ii) a certificate for G as defined in part a, and decides if the certificate is valid for G.

(c) Provide size parameters that may be used the measure the size of an instance of `Graph Matching`.

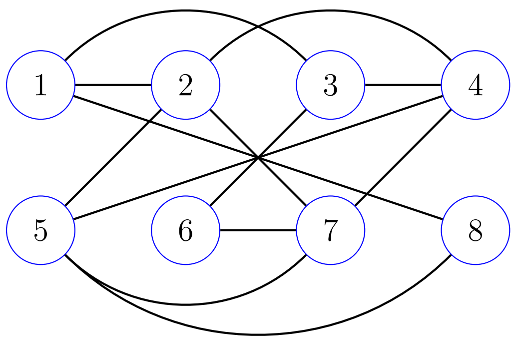
(d) Use the size parameters from part c to describe the running time of your verifier from part b. Defend your answer in relation to the algorithm you provided for the verifier.

Solutions

LO1. Answer the following.

(a) See Definition 2.1 of Map Reducibility Lecture.

(b) $f(G) = \overline{G}$ is shown below.



(c) G's maximum independent set is $\{2, 4, 5, 7\}$ which is the maximum clique for $f(G) = \overline{G}$. Therefore, G and $\overline{G}$ have the same solution as is required for a mapping reduction.

LO2. An instance of the `Graph Matching` decision problem is a simple graph $G = (V, E)$, and the problem is to decide if G has a **matching**, i.e. a set $M \subseteq E$ of $|V|/2$ edges for which every vertex of G is incident with exactly one edge $e \in M$.

(a) For a given instance $G = (V, E)$ of `Graph Matching` describe a certificate in relation to G.

Solution. A certificate for G is a subset $M \subseteq E$ having size $\lfloor n/2 \rfloor$, where $n = |V|$.

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance $G = (V, E)$, ii) a certificate for G as defined in part a, and decides if the certificate is valid for G.

Solution.

If $|V|$ is odd, then return 0.
Create a hash table T of size n.
For each $e = (a, b) \in M$,
  If $a \in T \ \vee \ b \in T$, then return 0.
  //Some vertex is incident with more than one edge of M
  Add a and b to T.
Return 1.

(c) Provide size parameters that may be used the measure the size of an instance of `Graph Matching`.

Solution. $m = |E|$, $n = |V|$.

(d) Building the hash table T requires $O(n)$ steps (assuming a table size of n and that it takes one step to allocate one cell of the table as is the case with some models of computation). Iterating through each of the $n/2$ edges of M also requires $O(n)$ steps because, for $e = (a, b)$, checking if either $a \in T$ or $b \in T$ requires $O(1)$ steps, as does adding those vertices to T. Therefore, the verifier requires $O(n)$ steps.