CECS 329, Learning Outcome Assessment 11, November 30th, Fall 2023, Dr. Ebert

Problems

LO7. Answer/solve the following.

(a) When universal program P_U simulates the computation $P_x(y)$, how is y used by P_U as part of the simulation?

(b) A universal program P_U is simulating a program that has 93 instructions and whose Gödel number is

$$x = 2^{31} + 2^{32} + 2^{44} + 2^{68} + 2^{256} + \cdots + 2^{c_{93}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^5 + 2^9 + 2^{11} + 2^{18} + 2^{21} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO8. Answer the following.

(a) Define what it means to be a positive instance of decision problem `Self Accept`. Hint: "Gödel number x is a positive instance of `Self Accept` iff ____."

(b) The goal is to show that `Self Accept` is undecidable. We assume it is decidable by assuming that its characteristic function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a positive instance of Self Accept} \\ 0 & \text{if } x \text{ is a negative instance of Self Accept} \end{cases}$$

is total computable. Provide the definition for how to compute the "antagonist" function $g(x)$ based on the value of $f(x)$.

(c) By writing the values of $g(0), g(1), \ldots$ in the appropriate table cells (next to the number in that cell), verify that function g is different from each computable function ϕ_i, $i = 0, 1, \ldots$, Why does this create a contradiction?

index\input x	0	1	2	$\cdots$	i	$\cdots$	**self accepting?**
$\phi_0(x)$	$\uparrow$	12	7	$\cdots$	$\uparrow$	$\cdots$	no
$\phi_1(x)$	8	87	36	$\cdots$	96	$\cdots$	yes
$\phi_2(x)$	7	5	0	$\cdots$	$\uparrow$	$\cdots$	yes
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
$\phi_i(x)$	0	32	65	$\cdots$	$\uparrow$	$\cdots$	no
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$

LO9. An instance of the decision problem `Total` is a Gödel number x, and the problem is to decide if program P_x halts on every input. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } P_x \text{ halts on every input} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

   i. $x = e_1$, where e_1 is the Gödel number of the program $P_{e_1} = S(1), S(1), T(1,2), J(1,2,2)$. Hint: $T(i,j)$ means $R_j \leftarrow R_i$.

   ii. $x = e_2$, where e_2 is the Gödel number of the program $P_{e_2} = S(1), S(1), J(1,2,1)$.

   iii. $x = e_3$, where e_3 is the Gödel number of the program that computes $g(x)$ (assuming that $g(x)$ is URM computable).

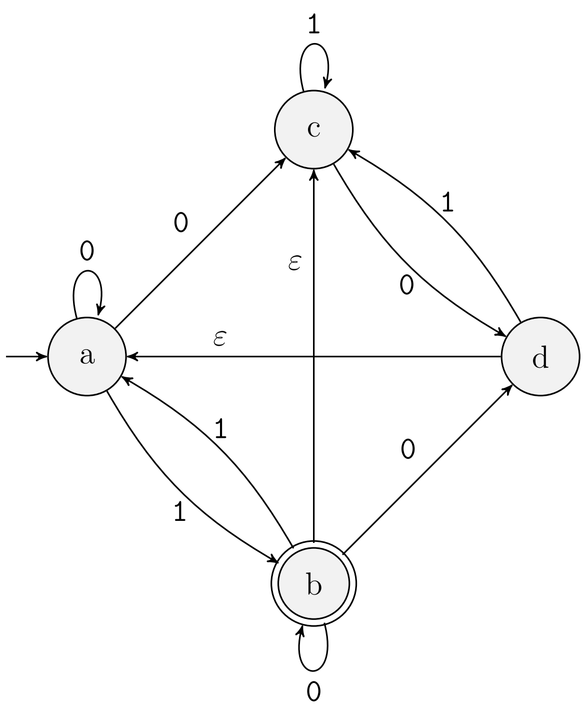
(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not P_x halts on all inputs. Do this by writing a program P that uses g and makes use of the `self` programming concept. Then use a proof by cases to show that P creates a contradiction.

LO10. Solve the following.

(a) Provide the state diagram for a DFA M that accepts a binary word w iff either w has no 0's, or every 0 of w is *immediately* followed by at least two consecutive 1's.

(b) Show the computation of M on inputs i) $w_1 = 10110111$ and ii) $w_2 = 01101101$.

LO11. Do the following for the NFA N whose state diagram is shown below.



(a) Provide a table that represents N's δ transition function.

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

(c) Show the computation of M on input $w = 11001$.