CECS 329, Solutions to Learning Outcome Assessment 11, November 30th, Fall 2023, Dr. Ebert

Problems

LO7. Answer/solve the following.

(a) When universal program P_U simulates the computation $P_x(y)$, how is y used by P_U as part of the simulation?

Solution. P_U uses y to create the initial configuration of the computation of $P_x(y)$.

(b) A universal program P_U is simulating a program that has 93 instructions and whose Gödel number is

$$x = 2^{31} + 2^{32} + 2^{44} + 2^{68} + 2^{256} + \cdots + 2^{c_{93}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^5 + 2^9 + 2^{11} + 2^{18} + 2^{21} - 1,$$

then provide the next configuration of the computation *and* its encoding.

Solution. We have

$$c = \tau^{-1}(\sigma) = (5, 3, 1, 6, 2).$$

Also, $\beta(I_2) = 32 - 31 - 1 = 0$ and $0 \bmod 4 = 0$ implies that I_2 is the Zero instruction $Z(1)$, since $0/4 = 0$. Thus,

$$c_{\text{next}} = (0, 3, 1, 6, 3)$$

and

$$\tau(c_{\text{next}}) = 2^0 + 2^4 + 2^6 + 2^{13} + 2^{17} - 1,$$

LO8. Answer the following.

(a) Define what it means to be a positive instance of decision problem `Self Accept`. Hint: "Gödel number x is a positive instance of `Self Accept` iff ______."

Solution. Gödel number x is a positive instance of `Self Accept` iff $P_x(x)\downarrow$, meaning P halts on its own Gödel number.

(b) The goal is to show that `Self Accept` is undecidable. We assume it is decidable by assuming that its characteristic function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a positive instance of } \texttt{Self Accept} \\ 0 & \text{if } x \text{ is a negative instance of } \texttt{Self Accept} \end{cases}$$

is total computable. Provide the definition for how to compute the "antagonist" function $g(x)$ based on the value of $f(x)$.

Solution. See Section 3.1 of the Undecidability and Diagonalization Method lecture.

(c) By writing the values of $g(0), g(1), \ldots$ in the appropriate table cells (next to the number in that cell), verify that function g is different from each computable function ϕ_i, $i = 0, 1, \ldots$, Why does this create a contradiction?

Solution.

index\input x	0	1	2	$\cdots$	i	$\cdots$	**self accepting?**
$\phi_0(x)$	$\uparrow \to 0$	12	7	$\cdots$	$\uparrow$	$\cdots$	no
$\phi_1(x)$	8	$87 \to \uparrow$	36	$\cdots$	96	$\cdots$	yes
$\phi_2(x)$	7	5	$0 \to \uparrow$	$\cdots$	$\uparrow$	$\cdots$	yes
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
$\phi_i(x)$	0	32	65	$\cdots$	$\uparrow \to 0$	$\cdots$	no
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$

Function g is assumed computable yet, from the table, is different from each computable function, a contradiction. Conclusion: f's characteristic function is not total computable and so `Self Accept` is undecidable.

LO9. An instance of the decision problem `Total` is a Gödel number x, and the problem is to decide if program P_x halts on every input. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } P_x \text{ halts on every input} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where e_1 is the Gödel number of the program $P_{e_1} = S(1), S(1), T(1, 2), J(1, 2, 2)$. Hint: $T(i, j)$ means $R_j \leftarrow R_i$.

Solution. $g(e_1) = 0$ since P_{e_1} never halts on any input.

ii. $x = e_2$, where e_2 is the Gödel number of the program $P_{e_2} = S(1), S(1), J(1, 2, 1)$.

Solution. $g(e_2) = 1$ since $R_1 \neq R_2$ when instruction $J(1, 2, 1)$ is executed.

iii. $x = e_3$, where e_3 is the Gödel number of the program that computes $g(x)$ (assuming that $g(x)$ is URM computable).

Solution. $g(e_3) = 1$ since g is assumed total computable.

(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not P_x halts on all inputs. Do this by writing a program P that uses g and makes use of the `self` programming concept. Then use a proof by cases to show that P creates a contradiction.

Solution.

Program P

Input y.

If $g(\text{self}) = 1$, then loop forever.

Else return 0.

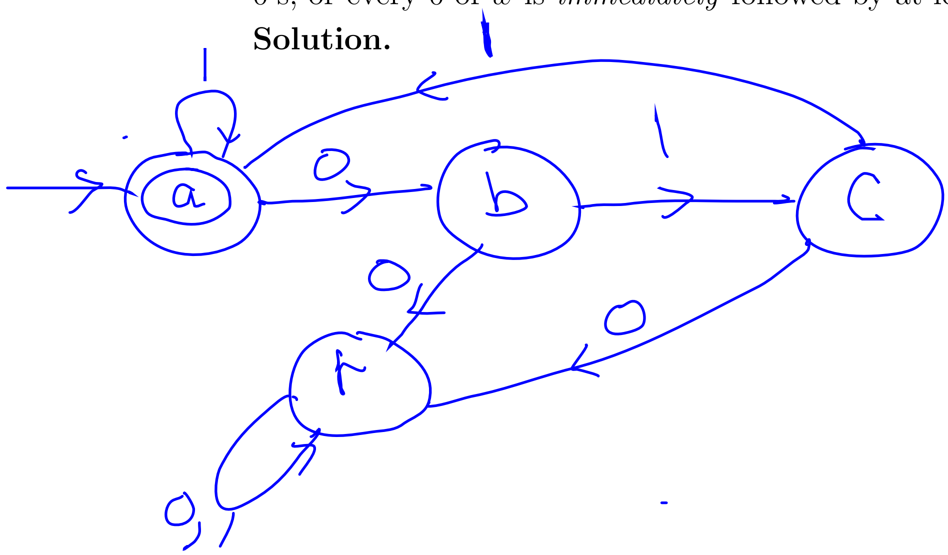
Case 1: $g(\text{self}) = 1$. Then P halts on all inputs, but P loops forever, a contradiction.

Case 2: $g(\text{self}) = 0$. Then P should not halt on all inputs, yet, for each input y, P outputs 0, a contradiction.

LO10. Solve the following.

(a) Provide the state diagram for a DFA M that accepts a binary word w iff either w has no 0's, or every 0 of w is *immediately* followed by at least two consecutive 1's.

Solution.



(b) Show the computation of M on inputs i) $w_1 = 10110111$ and ii) $w_2 = 01101101$.

Solution.

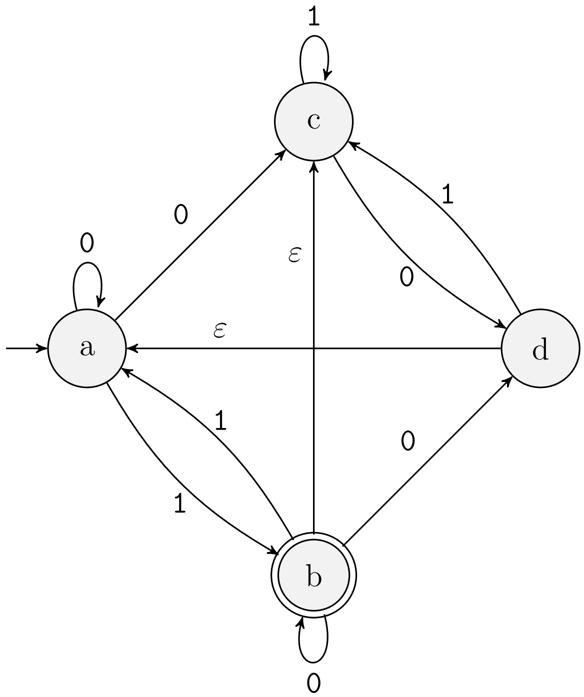
1 0 1 1 0 1 1 1

a a a b c a b c $\boxed{a}$ ⇒ accept

0 1 1 0 1 1 0 1

a b c a b c a b $\boxed{c}$ ⇒ reject

LO11. Do the following for the NFA N whose state diagram is shown below.



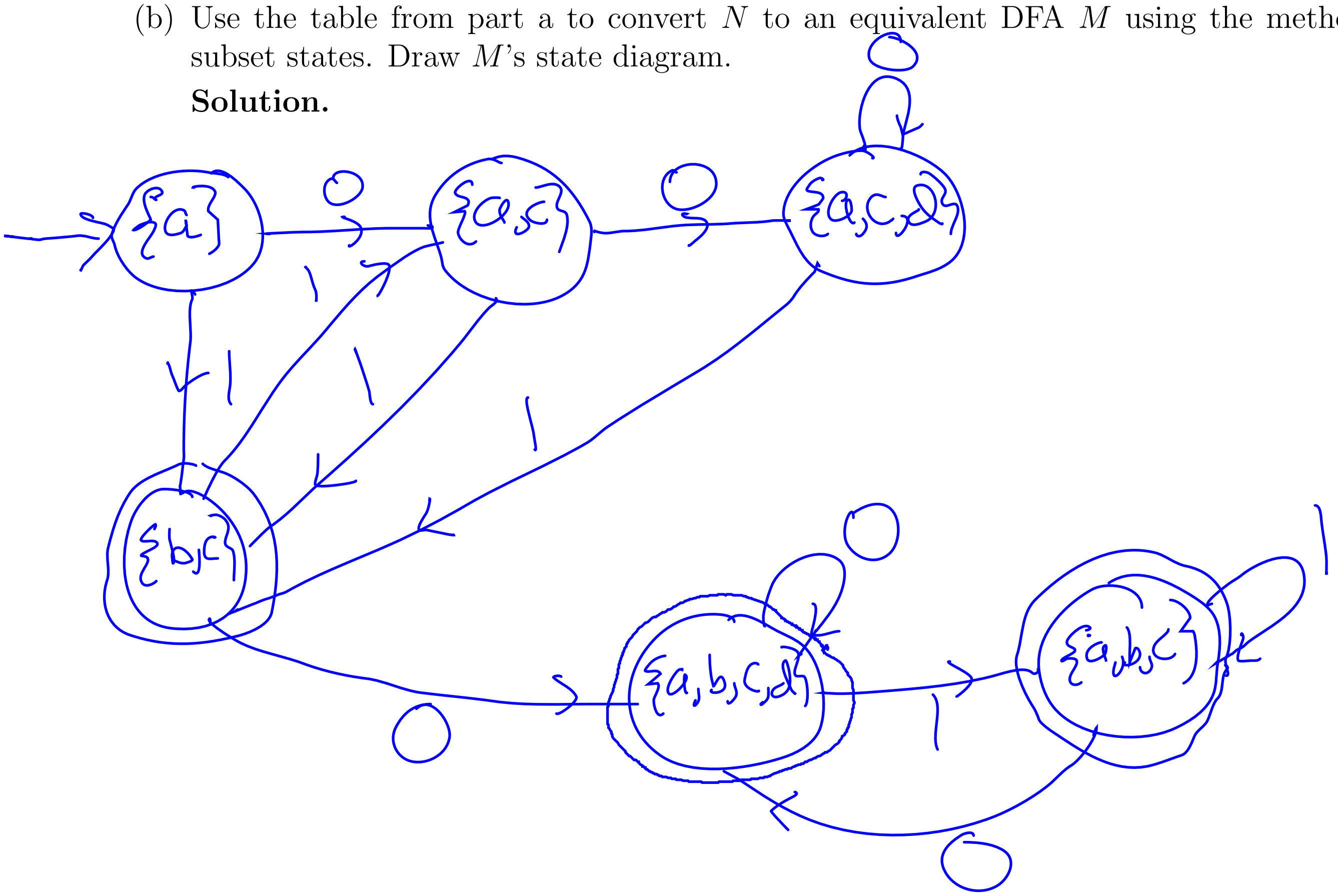
(a) Provide a table that represents N's δ transition function.

Solution.

δ ($Q \backslash \Sigma$)	0	1
a	$\{a, c\}$	$\{b, c\}$
b	$\{a, b, c, d\}$	$\{a\}$
c	$\{a, d\}$	$\{c\}$
d	$\emptyset$	$\{c\}$

(b) Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

Solution.



(c) Show the computation of M on input $w = 11001$.

$$\{a\} \xrightarrow{1} \{b,c\} \xrightarrow{1} \{a,c\} \xrightarrow{0} \{a,c,d\} \xrightarrow{0} \{a,c,d\} \xrightarrow{1} \boxed{\{b,c\}}$$

$\Downarrow$

accept