CECS 329, Learning Outcome Assessment 10, November 16th, Fall 2023, Dr. Ebert

Problems

LO6. Solve the following problems.

(a) Compute the Gödel number for program $P = S(5), Z(3), J(2, 1, 3), T(1, 2)$. Write your answer as a sum of powers of two minus 1 (see part b for an example). Show all work.

(b) Provide the URM program P whose Gödel number equals

$$2^{22} + 2^{40} + 2^{84} + 2^{113} - 1.$$

Show all work.

LO7. Answer/solve the following.

(a) When simulating the computation $P_x(y)$, why is it necessary for the universal program P_U to compute the maximum register index used by program P_x?

(b) A universal program P_U is simulating a program that has 93 instructions and whose Gödel number is

$$x = 2^{29} + 2^{73} + 2^{117} + 2^{149} + 2^{168} + \cdots + 2^{c_{93}} - 1.$$

If the current configuration of the computation of P_x on some input has encoding

$$\sigma = 2^5 + 2^{13} + 2^{17} + 2^{18} + 2^{20} - 1,$$

then provide the next configuration of the computation *and* its encoding.

LO8. Answer the following.

(a) One application of the diagonalization method is to prove that the set of partial unary functions cannot be placed in an infinite list, such as $f_0, f_1, f_2, \ldots$. This is done by assuming such a list of all functions exists and defining a partial unary function $g(x)$ that is different from every function in the list. Provide a definition for $g(x)$ that accomplishes this.

(b) Using your definition from part a and writing the values of $g(0), g(1), \ldots$ in the appropriate table cells (next to the number in that cell), verify that function g is different from each function f_i, $i = 0, 1, \ldots$. Conclude that the list does *not* include all partial unary functions.

index\input x	0	1	2	$\cdots$	i	$\cdots$
$f_0(x)$	$\uparrow$	12	7	$\cdots$	$\uparrow$	$\cdots$
$f_1(x)$	8	87	36	$\cdots$	96	$\cdots$
$f_2(x)$	7	5	0	$\cdots$	$\uparrow$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$f_i(x)$	0	32	65	$\cdots$	$\uparrow$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

(c) By the diagonalization proof provided, it follows that the set of partial unary functions is too large to be placed in an infinite list. How does this prove that there are some partial unary functions that are *not* URM-computable?

LO9. An instance of the decision problem `Even Range` is a Gödel number x, and the problem is to decide if function ϕ_x has a non-empty range and whose members are (not necessarily all the) even numbers. Consider the function

$$g(x) = \begin{cases} 1 & \text{if } \phi_x \text{ has a non-empty range of even numbers} \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate $g(x)$ for each of the following Gödel number's x. Note: 2 out of 3 correct is considered passing. **Justify your answers**.

i. $x = e_1$, where e_1 is the Gödel number of the program that computes the function $\phi_{e_1}(y) = 4y^2 + 3y$.

ii. $x = e_2$, where e_2 is the Gödel number of the program that computes the function $\phi_{e_2}(y) = 6$.

iii. $x = e_3$, where e_3 is the Gödel number of the program that computes $g(x)$ (assuming that $g(x)$ is URM computable).

(b) Prove that $g(x)$ is not URM computable. In other words, there is no URM program that, on input x, always halts and either outputs 1 or 0, depending on whether or not ϕ_x has a non-empty even range. Do this by writing a program P that uses g and makes use of the `self` programming concept. Then show how P creates a contradiction.

LO10. Solve the following.

(a) Provide the state diagram for a DFA M that accepts *all* binary words, *except* those that end with 010.

(b) Show the computation of M on inputs i) $w_1 = 110101$ and ii) $w_2 = 011010$.