

3.20. The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results are shown in the following table.

Circuit Type		Response Time			
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

(a) Test the hypothesis that the three circuit types have the same response time. Use $\alpha = 0.01$.

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_1 : \text{at least one mean is different}$$

From the SAS output below, the calculated F statistic is $F=16.08$ with 2 and 12 degrees of freedoms. The corresponding P-value is 0.0004 which is less than $\alpha = 0.01$, so there is at least one circuit type that is different.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	543.6000000	271.8000000	16.08	0.0004
Error	12	202.8000000	16.9000000		
Corrected Total	14	746.4000000			

Note: For the purpose of preparing the Comp, please also try to calculate Sum of squares using a calculator.

(b) Use Tukey's test to compare pairs of treatment means. Use $\alpha = 0.01$.

$$MSD = \frac{1}{\sqrt{2}} q_{0.01,3,12} \sqrt{\frac{2MSE}{n}} = \frac{1}{\sqrt{2}} (5.04589) \sqrt{\frac{2(16.9)}{5}} = 9.2768$$

$$1 \text{ vs. } 2: |10.8-22.2| = 11.4 > MSD$$

$$1 \text{ vs. } 3: |10.8-8.4| = 2.4 < MSD$$

$$2 \text{ vs. } 3: |22.2-8.4| = 13.8 > MSD$$

1 and 2 are different. 2 and 3 are different.

Notice that the results indicate that the mean of treatment 2 differs from the means of both treatments 1 and 3, and that the means for treatments 1 and 3 are the same. The result is also confirmed in the SAS output below.

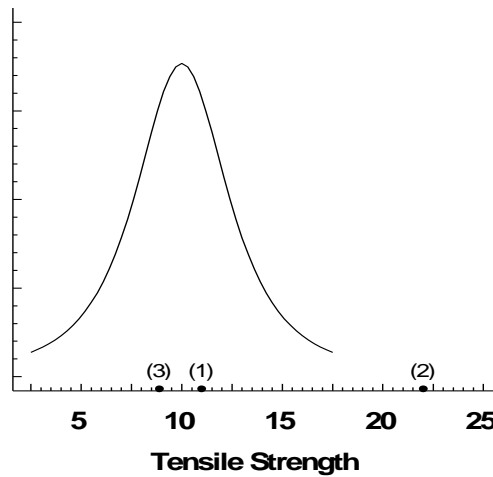
Alpha	0.01
Error Degrees of Freedom	12
Error Mean Square	16.9
Critical Value of Studentized Range	5.04589
Minimum Significant Difference	9.2768

Means with the same letter are not significantly different.			
Tukey Grouping	Mean	N	type
A	22.200	5	2
B	10.800	5	1
B			
B	8.400	5	3

- (c) Use the graphical procedure in Section 3.5.3 to compare the treatment means. What conclusions can you draw? How do they compare with the conclusions from part (a).

The scaled- t plot agrees with part (b). In this case, the large difference between the mean of treatment 2 and the other two treatments is very obvious.

Scaled t Distribution



- (d) Construct a set of orthogonal contrasts, assuming that at the outset of the experiment you suspected the response time of circuit type 2 to be different from the other two.

$$H_0 = \mu_1 - 2\mu_2 + \mu_3 = 0$$

$$H_1 = \mu_1 - 2\mu_2 + \mu_3 \neq 0$$

$$C_1 = y_1 - 2y_2 + y_3$$

$$C_1 = 54 - 2(111) + 42 = -126$$

$$SS_{C_1} = \frac{(-126)^2}{5(6)} = 529.2$$

$$F_{C_1} = \frac{529.2}{16.9} = 31.31$$

Type 2 differs from the average of type 1 and type 3.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Type 2 vs Type 1 & 3	1	529.2000000	529.2000000	31.31	0.0001

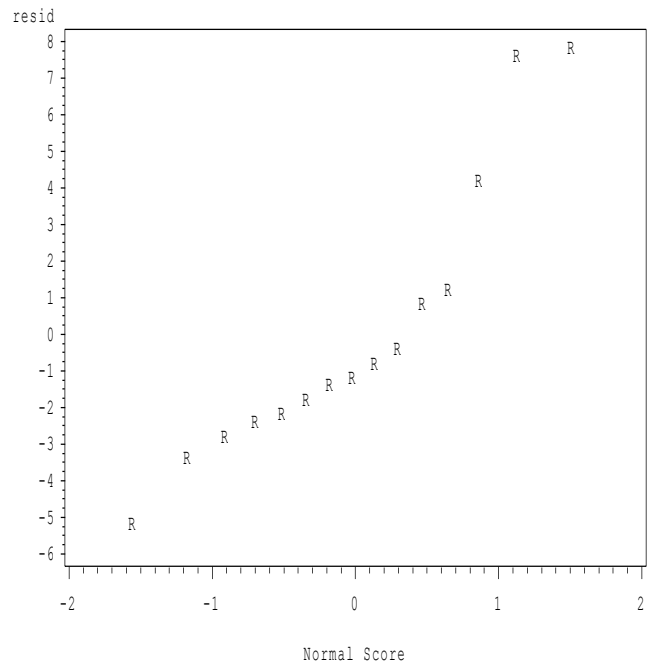
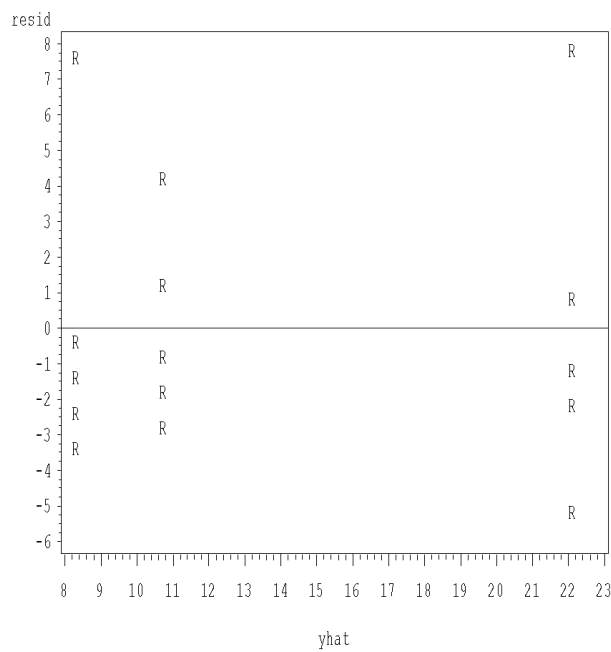
(e) If you were a design engineer and you wished to minimize the response time, which circuit type would you select?

Either type 1 or type 3 as they are not different from each other and have the lowest response time.

(f) Analyze the residuals from this experiment. Are the basic analysis of variance assumptions satisfied?

The residual plot (left) shows some evidence of unequal variances, however, the Levene's test for the equal variance in the table below shows that the equal variance assumption is valid (P-value = .6145).

The normal probability plot (right) has some points that do not lie along the line in the upper region. This may indicate potential outliers in the data; however, the standardized residuals at those points are still within ± 2 . We can conclude that the normal and equal variance assumptions on the model errors are satisfied.



Levene's Test for Homogeneity of time Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
type	2	437.2	218.6	0.51	0.6145
Error	12	5172.0	431.0		

<SAS code used>

```
DM "output;clear;log;clear";
Options pagesize=45 linesize=80 PageNo=1 NoDate;

ODS RTF File="C:\teaching\stat530\pr3_20.rtf";
ODS Listing Close;
Data circuit;
input type time;
Datalines;
1 9
1 12
1 10
1 8
1 15
2 20
2 21
2 23
2 17
2 30
3 6
3 5
3 8
3 16
3 7
;
Proc GLM;
class type;
model time=type;
means type / hovtest tukey scheffe bon alpha=0.01;
contrast 'Type 2 vs Type 1 & 3' type 1 -2 1;
estimate 'Type 2 vs Type 1 & 3' type 1 -2 1;
output out=circuit p=yhat r=resid; run;
Proc gplot;
plot yhat*power ;
plot resid*yhat='R' / vref=0 ; run;
proc univariate normal plot;
var resid; run;
proc rank normal=vw; /* Computing ranked normal scores by residuals*/
var resid;
ranks nscore; run;
proc gplot ;
plot resid*nscore='R'; /*plotting ranked residual vs. normal score*/
label nscore='Normal Score'; run;
proc corr; /* calculate correlation efficient btwn resid and nscore */
var resid nscore; run;

ODS Listing;
ODS RTF Close;
```