

Commutative Diagrams in the Fine Arts

Karl Heinrich Hofmann

Almost imperceptibly, mathematics has emerged into public view in works of fiction, theater, and cinematography, and people at large suddenly find mathematicians to be interesting characters. Like a number of my colleagues, I have been to see the Broadway play *Proof* [1], in which all of the characters are attached to mathematics in one way or another. Its author, David Auburn, received a Pulitzer Prize and a Best Play Tony Award [7]. His play was amply reviewed in mathematics journals, indeed twice in the *Notices* [2], [11] (see also [3], [18]). Remarkable popular acclaim keeps it running on Broadway, and it is now appearing on stages abroad [3]. It is undoubtedly good for mathematics to appear as a normal human endeavour, but in my mind I see the question of *authenticity* arise instantly whenever I encounter cross-cultural links between the humanities and mathematics. In *Proof*, we listen in on the following dialogue between Hal, a young mathematics Ph.D., and the protagonist Catherine (Act I, Scene 3, [1], p. 34):

Hal: [...] Mathematicians are insane. I went to this conference [...] last fall. I'm young, right? I'm in shape. I thought I could hang out with the big boys. Wrong. I've never been so exhausted in my life. Forty-eight straight hours of partying, drinking, drugs, papers, lectures...

Catherine: Drugs?

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Hal: Yeah. Amphetamines mostly.[...] Some of the older guys are really hooked. [...] They think math's a young man's game. Speed keeps them racing, makes them feel sharp. There's this fear that your creativity peaks around twenty-three and it's all downhill from there. Once you hit fifty it's over, you might as well teach high school.

I guess I needed to be told that for me, at least, it's been all “over” for the last twenty years. I suppose also that in all probability, in all these and the preceding years, I have attended the wrong meetings. Little consolation that younger mathematicians expressed surprise about this kind of “proof” as well [18]!

If a cultural relationship between mathematics and the stage has its tenuous points even under the best of circumstances, then perhaps relief is in sight when we discover mathematics as a core topic in the fine arts.

Not long ago, *New York Arts Magazine* published a feature essay [9] in which was discussed a segment of the remarkable work of the French-American artist Bernar Venet, in which he recycles the typography of mathematical formulae into a novel genre of conceptual paintings of monumental proportions. The essay concludes as follows:

“The Greek ‘mathemat’ means lesson or learning, but Venet’s use of color as the ground for his mathematical “figures” undermines their didactic look. [...] We are no longer afraid to be ignorant, for the color allows us to embrace our ignorance as the way to the emotional truth. [...] the alienness of the mathematics becomes an entry into the emotional depths. What emotional truth, what

emotional truth? I suggest it is a sexual truth and depth [...], which at its deepest establishes an erotic relationship with the spectator. And which in itself re-enacts the sexual union of opposites. I suggest that Venet's wall paintings do so, without showing its consummation. They are profoundly sexual in import, on a grand scale that masks their poignancy" [9].

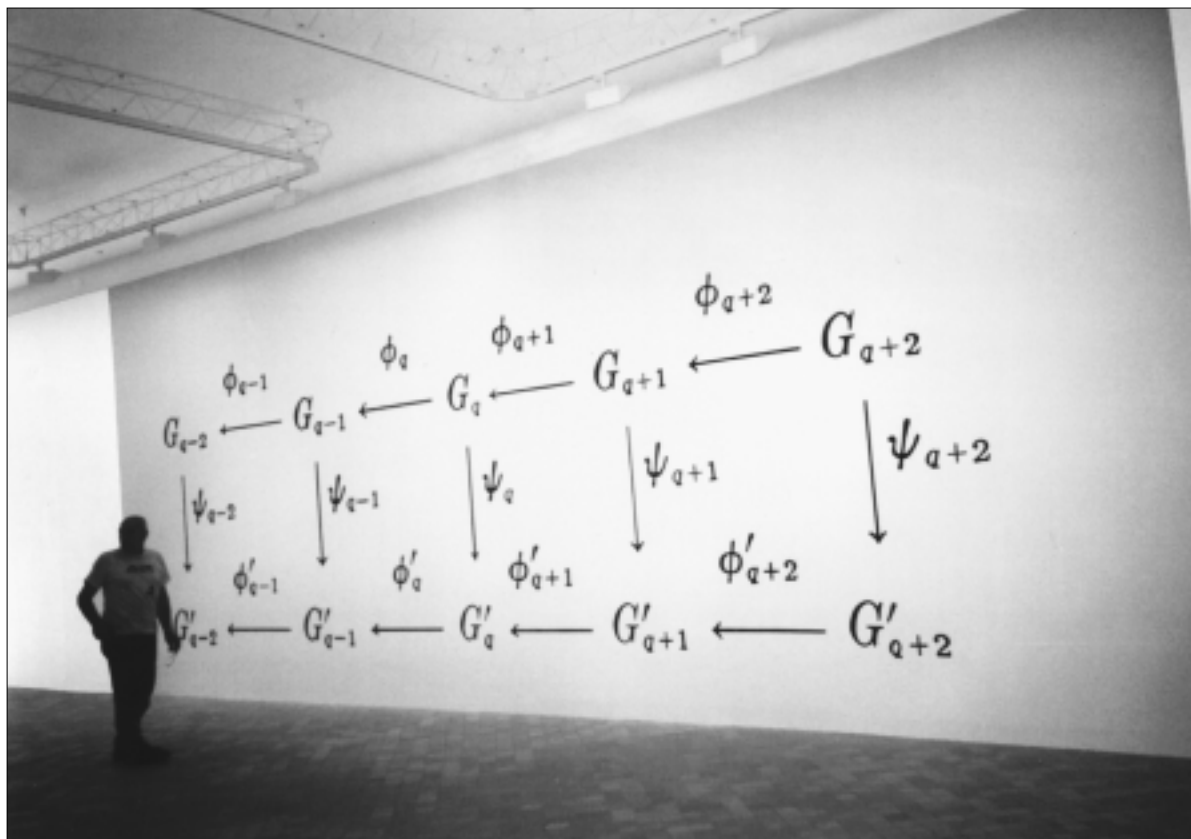
Having done mathematics more than anything else during these past fifty years while staying close to the fine arts by inclination, I am again asking myself: Could it be possible that I have missed out on something?

When I visit the Mathematisches Forschungsinstitut in Oberwolfach for a conference, barely a few hours pass before some mathematician occupies the music room, playing the piano or orchestrating some chamber music with like-minded colleagues. "Scratch a mathematician; find a musician," as the saying goes. Musical and mathematical talents appear to cohabitate comfortably. The concert goers and opera buffs among us appear to be legion. But our cohort of museum visitors, gallery hoppers, and artists is much smaller; yet it was distinctly present at various events during the International Congress of Mathematicians in Berlin in 1998. Perhaps the aesthetics of computer-generated

art has helped us to recruit art lovers among mathematicians. Remarkably, one of the featured speakers at the Berlin Congress was a writer and poet well known on the literary scene—certainly in Germany. Hans Magnus Enzensberger addressed the Congress in a speech entitled "Drawbridge Up", metaphorically referring to mathematics as a castle separated from the humanities and arts by deep moats and dysfunctional bridges. The bilingual publication of this address in German and English brilliantly pleads for the central place of mathematics within the panoply of cultures [4] and is a pleasure to read for all mathematicians; after C. P. Snow's "Two Cultures" this is surely one of the grand essays on the integration of mathematics in the universe of creative intellectual activity. Regarding the relations, more specifically, between *mathematics and the fine arts*, until we have a grand essay by some articulate member of the writing guild, we have to content ourselves in the meantime with finding and commenting on artists that allow us to study this relationship in concrete cases. Such an artist is Bernar Venet whose versatile oeuvre is impressive in its wide horizon, encompassing sculpture [10], painting [9], poetry [15]. His sculptures are publicly placed; one of the steel constructs from his series entitled *Indeterminate*

Photographs of paintings courtesy of Bernar Venet.





Lines [10] is positioned on the MIT campus in Cambridge (Massachusetts) in front of Dewey Library: *Two Indeterminate Lines* [17]. The very compact excerpt of Venet's curriculum vitae, which is appended, gives only a very incomplete picture of this artist's creative horizon.

Naturally, as mathematicians we are particularly interested in that very recent portion of his work which appears to link very closely with mathematics and which generates such orgiastic sentiments in the souls of trained art critics. But professional mathematicians should not be led astray by the professional arts commentators. We as mathematicians are in an excellent position to appreciate and experience straightforwardly the outside aesthetics of Venet's mathematical conceptual art as well as its intrinsic quality.

What, concretely, is Venet doing? He creates wall paintings of imposing scale with radiant coloration under graphical structures that are taken from mathematical formula language. The artist lifts from mathematical texts and monographs items like formulae or illustrations and transfers enlarged copies of these in black, that is, in the color of the original printing ink, onto luminously monochromatic surfaces, generating, in the process, paintings of a monumental format, filling entire wall segments in an exhibition hall. Well centered as they were in a rectangular space allotted to them in the book, the formulae now fill the rectangle bounded by the frame of the picture. The deliberate positioning of a formula in a book originally served the

purpose of transmitting scientific information for which the typesetter aspires to optimal clarity and legibility. Prominent examples used by Venet are homological diagrams (for him, the bigger the better), vector fields in the plane, matrix calculations, level-line diagrams. The sources tapped by Venet are (surely in addition to others) [19], [20], [22], [23], [24], [25]. Among the space filling formulae, the ones most striking to the mathematician are probably the homological commutative diagrams from Eilenberg-Steenrod's classic [20]. Even though other illustrations from mathematical texts are taken as well, the preponderance of designs used appear to be of a typographical nature.

Mathematicians are likely to react and respond immediately; outsiders are probably surprised if not stunned by the artist's proposition that tokens of a highly specialized technical language are to be used as building blocks of a new artistic expression. The element of surprise is calculated [15]: "Il n'y a qu'un moyen de faire avancer l'art. C'est de donner tort à l'art déjà constitué" (Bernar Venet 1999). ["There is only one way to make art advance: to put existing art into the wrong."] In Venet's work, mathematical typography is recognized as its own graphical and architectural structure, utilized and elevated artistically in a twofold fashion: First, by the brilliant monochromatic backgrounds, and second, by the monumental format. The ostentatiously colorful character of these works notwithstanding, they are largely graphical; it appears appropriate to describe them as "wall graphics".

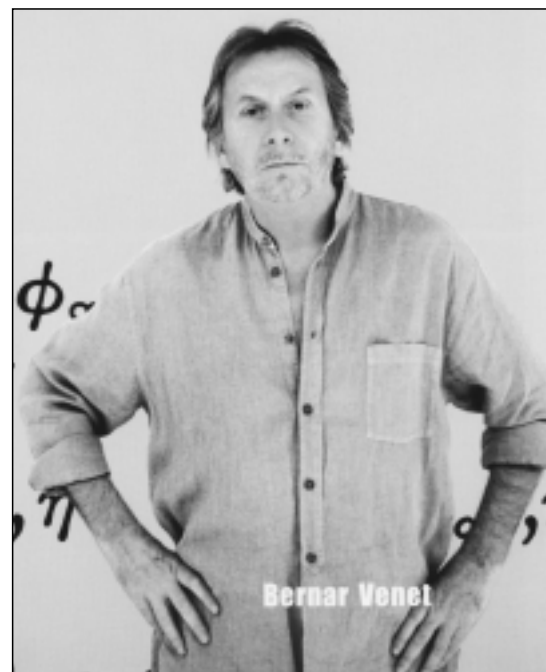
For two decades, working mathematicians have been in a position to appreciate typography as an artistic endeavor. Donald Knuth [8] first empowered us—and by now forced us—to typeset our own texts and create the typography of our formulae *with our own hands*. While it is true that we are doing this with our fingers on the keyboards rather than by assembling lead cast letters, Knuth has shaped his programming language \TeX so that it faithfully emulates the original craft. The title of Spivak’s handbook on the use of $\mathcal{A}_{\mathcal{M}\mathcal{S}}\text{\TeX}$ is *The Joy of \TeX* . Most of us have realized the challenges of typography and experienced the satisfaction arising out of the accomplishment of having found—for problems involving the typesetting of a formula, the arrangement of a diagram, or the organization of a printed page—a solution that is particularly pleasing to the eye as well as clear in exposition. Having created a fine typographical product thus adds extra satisfaction to the pleasure of having found a mathematically aesthetic result, having proved it, and having presented it in a stylistically elegant fashion. Mathematicians, therefore, know of the experience brought about by an activity recognized by the world at large as artistic. The artist and art theoretician Joseph Beuys vigorously proposed the theme that each person, by virtue of one’s own creativity, is an artist (see [5]), and I suggest that mathematicians, quite apart from graphics and typography, are excellently qualified to understand this point. Hans Magnus Enzensberger, referring to Godfrey Harold Hardy’s *A Mathematician’s Apology*, expresses this sentiment:

“The autonomy Hardy requires for his basic research finds its counterpart in the arts, and it is no coincidence that most mathematicians are thoroughly comfortable with aesthetic criteria—a proof needs not just to be conclusive; the mathematician aspires to ‘elegance’. The word expresses a quite particular aesthetic sensibility that has characterized the mathematical enterprise since its earliest beginnings” [4, p. 25].

Venet elevates the tokens of the mathematicians’ language to a form of art in a very explicit fashion. To a mathematician it appears that he is partly motivated by an artist’s desire to make the mathematicians’ infatuation with “elegance”—certainly an aesthetic category—manifest for the layman.

If this all sounds a bit aloof, perhaps a chat about commutative diagrams makes the point, because some of us have struggled with them in our publications and because Venet chose to feature them prominently in his wall graphics. Mac Lane’s *Fundamental Theorem* says: *Every diagram is commutative*. To anyone who doubted that, Mac Lane proved it in the form of his coherence theorems [21]. They are complicated enough. Experience teaches us that, heuristically, a “natural” diagram is likely to be commutative. The suggestion that emerges from

the diagrams “formulated” by Venet seems to confirm this—contrary to our having often experienced how difficult it can be to actually verify a seemingly obvious commutativity. The commutativity of a diagram represents one or several equations—well known to the aficionados. Suppose we step back for a moment and take stock: Is it not remarkable that homological algebra and category theory have created, during the third quarter of the last century, a graphical and therefore visible expression for certain equations? The words “graph” and “diagram” still exhibit the Greek word for “writing”. On many occasions I have experienced the effort of organizing and drawing a supposedly commutative diagram on a piece of paper as stressful, and I have found that a good visualisation is often the essential step in the final proof of the equation in the background. The actual drawing of a commutative diagram by the working mathematician is an artistic activity even if not consciously performed as such.



Photograph courtesy of NY Arts Magazine.

Venet, the professional artist, may not be aware of the technicalities of the mathematics underlying the diagrams he uses, but he certainly senses that artistic element in their creation. He comments on his own work in a more rigorously purist mode: “I did not present Mathematics as Art: but Mathematics as such, merely for its own importance, for its own function. The Art existed only on a propositional and conceptual level” [14]. Whatever his artistic message to the general public may be—mathematicians at any rate are reminded at this juncture of one of the many points of contact between the mathematical and the artistic aspects of

their own work, mathematicians as artists, as Beuys had postulated. Venet claims to present “mathematics as such.” We concede, as mathematicians, that he attains an uncontested degree of authenticity by copying material from its scholarly environment without modifying or adulterating it, and thus maintains what we call “truth”.

Suppose that, in Venet’s monumentally presented painting *Related to the Homology Simplicial Complexes*, in the place of $H_{q-1}(|L^{q-1}|)$ the artist had copied $H^{q-1}(|L^{q-1}|)$. The extraneous aesthetic quality of the wall graphic would not in the least be impaired. Most observers other than algebraic topologists would not even notice what would be in fact a mathematical absurdity; the substitute is a perfectly sensible expression for a perfectly meaningful cohomology group—but it has no place in a diagram of homology groups. Such a minute transformation in the graphical design does not affect the design nor its monumentality nor its coloration, and from a purely graphical point of view, it could still be enjoyed no less than a page of Chinese calligraphy might be enjoyed by a person who is unable to read it. The modification does, however, invalidate the mathematics. The art critic without mathematical training would still be enraptured by coloration and depth. But what about “truth”? The aesthetics of Venet’s mathematical wall graphics therefore inhabits two levels. I suggest that Venet’s use of mathematical material serves—quite apart from the external aesthetic charm—as a semiotic reference, as a signal pointing to creativity in an other than the purely artistic domain in which beauty expresses itself through the truth, coherence, and logical elegance of propositions and through nothing else. How should the artist otherwise make evident the parallels between logical and pictorial creativity than by amalgamating the expressive material of both? In a recent play [13], during a discourse on the relative virtues of scholarship and poetry, the author, Tom Stoppard, has one of his characters say (Act I, p. 37): “Taste is not knowledge. A scholar’s business is to add to what is known. That is all. But it is capable of giving the very greatest satisfaction, because knowledge is good. It does not have to look good or sound good or even do good. It is good just by being knowledge. And the only thing that makes it knowledge is that it is true.” Venet’s recent paintings present us with a marvellous synthesis of scholarship and art along just this line.

Source Material

- [1] DAVID AUBURN, *Proof*, Faber and Faber, New York, London, ISBN 0-571-19997-6, 2001, 83 pp.
- [2] DAVE BAYER, *Proof*, Review of *Proof*, a play by David Auburn, *Notices Amer. Math. Soc.* **47** (2000), 1082–4.
- [3] REINHARD DIESTEL, *Proof/Der Beweis*—von David Auburn, Eine sehenswerte deutsche Uraufführung an den Hamburger Kammerspielen, *DMV-Mitteilungen*

(*Mitteilungen der Deutschen Mathematiker-Vereinigung*) 4–2001, 54–5.

- [4] HANS MAGNUS ENZENSBERGER, *Drawbridge Up—Zugbrücke außer Betrieb*, A K Peters, Natick, MA, 1999, German and English translation by Tom Artin, 47 pp.
- [5] KARL HEINRICH HOFMANN, Ein Mathematisches Fundstück, *DMV-Mitteilungen* 2–1999, 62.
- [6] ———, Die Ästhetik von Formeln: Bernar Venets Wandbilder, *DMV-Mitteilungen* 3–2001, 27–32.
- [7] ELAINE KEHOE, *Proof* wins Best Play Tony Award, *Notices Amer. Math. Soc.* **48** (2001), 839.
- [8] DONALD E. KNUTH, *The T_EX Book*, Addison Wesley, Reading, 1984ff.
- [9] DONALD KUSPIT, Bernar Venet’s New Wall Paintings, The Aesthetics of the Mathematical Formula or a Mathematical Formula for Aesthetics? *New York Arts Magazine* **5**, no. 9 (September 2000), 33–41.
- [10] CARTER RATCLIFF, Bernar Venet, *Sculpture Magazine* **18** (1999), No 2: <http://www.sculpture.org/documents/scmag99/march99/venet/venet.htm>.
- [11] MARK SAUL, The mathematician’s *Proof*, Review of *Proof*, a play by David Auburn, *Notices Amer. Math. Soc.* **47** (2000), 596–7.
- [12] MICHAEL D. SPIVAK, *The Joy of T_EX, A Gourmet Guide to Typesetting*, Amer. Math. Soc., Providence, 1982ff.
- [13] TOM STOPPARD, *The Invention of Love*, Faber and Faber, London 1997.
- [14] JAN VAN DER MARCK, *Bernar Venet, 1961–1970*, Éditions des Cahiers intempestatifs, 1999.
- [15] BERNAR VENET, *Apoétiques 1967–1998*, Musée d’Art Moderne et Contemporain (MAMCO), Genève, 1999, ISBN: 2-940159-12-2.
- [16] Web site of Musée d’Art Moderne et Contemporain (MAMCO), Genève: <http://www.mamco.ch/>.
- [17] SARAH H. WRIGHT, Campus sculptures brought to life by curator’s commentary, MIT Tech Talk August 2001: <http://web.mit.edu/newsoffice/tt/2001/aug29/arning.html>.
- [18] GÜNTER M. ZIEGLER, Pulitzer Preis für einen Beweis, *DMV-Mitteilungen* 2–2001, 54.

Sources for Venet’s Wall Paintings

- [19] ROBERT B. ASH, *Basic Probability Theory*, John Wiley, New York, 1970.
- [20] SAMUEL EILENBERG and NORMAN STEENROD, *Foundations of Algebraic Topology*, Princeton University Press, 1952.
- [21] SAUNDERS MAC LANE, Categorical algebra, *Bull. Amer. Math. Soc.* **71** (1965), 40–106.
- [22] JOHANN PFANZAGL, *Theory of Measurement*, Physica-Verlag Würzburg, 1973.
- [23] PAUL ROMAN, *Introduction to Quantum Field Theory*, Wiley, New York, etc., 1969.
- [24] CARL LUDWIG SIEGEL, *Topics in Complex Function Theory I–III*, Wiley Interscience, New York, 1969–73.
- [25] VAN COURT HARE JR., *Systems Analysis: A Diagnostic Approach*, Harcourt, New York, 1976.

Appendix: Excerpts from Venet’s Biography

1941. BERNAR VENET is born on April 20 at Château-Arnoux-Saint-Auban in the Alpes de Haute-Provence. At age eleven, he is invited to exhibit in the Salon de Peinture Péchiney in Paris.

- 1963–64. VENET establishes a studio in the old quarter of Nice. His first sculpture, *Coal Pile*, has no specific dimensions. The work is characterized by extremely restrained means. He participates in the Salon Comparaisons at the Museum of Modern Art, Paris. He exhibits alongside the New Realist and Pop Artists, despite the intentionally divergent nature of his cardboard reliefs.
1966. VENET takes his first trip to New York in April and May. He returns to Nice. In the course of his work he becomes aware of the objective aspect of blueprints and their semantic characteristics. He works intently on diagrams, creating his first monosemiotic works. In December, he decides to permanently settle in New York. Initially he lives in Arman's studio, 84 Walker Street, formerly Tinguely's studio.
- 1967–68. His conceptual work develops along logical lines. VENET frequents the mathematics and physics departments at Columbia University and befriends two researchers, JACK ULLMAN and MARTIN KRIEGER. He collaborates with the scientists from Columbia University to stage a performance at the Judson Church Theater in SoHo, New York. Two conceptual exhibitions take place at the Wide White Space Gallery and, along with Beuys and Broodthaers, at the Düsseldorf Kunsthalle. Works are bought by the Krefeld Museum, which offers to stage his first museum exhibition. The Museum of Modern Art, New York, acquires a VENET piece.
- 1969–71. Retrospectives are organized at the Krefeld Museum, Germany, and at the New York Cultural Center, New York.
- 1972–74. VENET returns to Paris. Teaches "Art and Art Theory" at the Sorbonne, Paris. Frequent lectures in France, England, Italy, Poland, and Belgium. His conceptual work is exhibited at the Institute of Contemporary Art, London. He represents France at the XIII São Paulo Biennale, Brazil, with Gottfried Honegger and François Morellet.
1976. VENET returns to New York in January and produces his first canvases from the series *Angles and Arcs*, a group of extremely restrained paintings of elementary geometrical figures. A retrospective of his conceptual works is shown at La Jolla Museum of Contemporary Art, La Jolla, California.
- 1977–78. VENET exhibits at Documenta VI, Kassel, Germany, and participates in the exhibit *From Nature to Art. From Art to Nature*, at the Venice Biennale, Italy.
1979. He begins the series of wood reliefs *Arcs, Angles, Diagonals*, creates the first *Indeterminate Line*, and starts work on steel sculptures composed of two arcs. VENET receives a grant from the National Endowment of the Arts. He develops the series of wood reliefs *Indeterminate Lines*.
1987. For the 750th anniversary of Berlin, the French Ministry of Foreign Affairs and Air France present the city with *Arc of 124.5°*. This sculpture measures 70 x 140 feet.
- 1988–89. VENET receives the Design Award for his sculpture in Norfolk, Virginia. He is commissioned to create the monumental *Two Indeterminate Lines* for the new La Défense business center on the edge of Paris and is awarded the Grand Prix des Arts de la Ville de Paris, by JACQUES CHIRAC, mayor of Paris.
1990. The monumental sculpture *Indeterminate Line* is inaugurated at Place de Bordeaux, Strasbourg, France.
1992. VENET travels to Japan for the inauguration of the sculpture at Adachi-ku, Tokyo.
1993. Retrospective exhibition at the Musée d'Art Moderne et d'Art Contemporain in Nice, France, which travels to the Wilhelm-Hack Museum in Ludwigshafen, Germany.
1994. From May through July, JACQUES CHIRAC, then the mayor of Paris, invites VENET to present twelve sculptures from his *Indeterminate Lines* series on the Champ de Mars.
1995. In the spring he travels to San Francisco for the inauguration of his monumental sculpture *Indeterminate Line*, at the Runnymede Sculpture Farm. In May, he inaugurates in Kowloon, Hong Kong, at the Museum of Modern Art, the first exhibition of the world tour of the works presented in 1994 at Champ de Mars. In June, he is the first artist to inaugurate the new Museum of Art in Shanghai. He starts to execute new reliefs in steel with an acetylene torch: *Indeterminate Area*.
- 1996, 1997. VENET is awarded the honor of Commandeur dans l'ordre des Arts et Lettres by the Minister of Culture in France and becomes a Member of the European Academy of Sciences and Arts in Salzburg, Austria.
1998. VENET travels to China and is invited by the mayor of Shanghai to participate in the Shanghai International Sculpture Symposium.
1999. Installation of a public sculpture in the City of Cologne, Germany, in honor of the G-8 Summit. Publication of his poetry *Apoétiques 1967–1998*, by the Musée d'Art Moderne et Contemporain, Geneva. Public commissions for the new university in Geneva.
- 2000, 2001. VENET develops a new series of wall paintings called *Major Equations*, which are exhibited at the Museu de Arte Moderna in Rio de Janeiro; at the Museu Brasileiro da Escultura in São Paulo; in the Teatro Nacional Brasília; at Centre d'Art Contemporain Georges Pompidou in Cajarç; and at MAMCO in Geneva. He exhibits the new paintings on canvas at the gallery Jérôme de Noirmont in Paris.

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Added in Proof, March 26, 2002. The remarks in the first paragraph of my article are now strengthened by the public success of the movie *A Beautiful Mind* (see *Notices* 4 (2002), 455–9) and by the acclaim heaped on it by the movie industry through the awarding of several Oscars, among them the top award for best film. —K. H. H.