Math 760 Day ? (2-7)

Uniqueness of prime decompositions

$M \cong P_1 \# \cdots \# P_k \# l(S^1 \times S^2)$

$\cong Q_1 \# \cdots \# Q_m \# n(S^1 \times S^2)$

Let S be a collection of 2-spheres s.t.

$M - S$ $\cup$ punctured P_i and punctured S^3

$M - T$ $\cup$ punctured Q_i and punctured S^3

Last time we can rechoose S s.t. $T \cap S = \emptyset$.

Then $M - (T \cup S)$ is the union of punctured P_i and punctured S^3

<u>and</u>

is the union of punctured Q_i and punctured S^3

Hence $k = m$ and $\{P_1, \ldots, P_k\} = \{Q_1, \ldots, Q_m\}$

To show $l = n$ note

$M \cong N \# l(S^1 \times S^2) \cong N \# n(S^1 \times S^2)$

$H_1(M) \cong H_1(N) \# \mathbb{Z}^l \cong H_1(N) \# \mathbb{Z}^n$

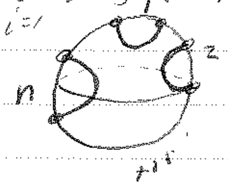
so $\boxed{l = n}$.

- define ambient isotopy.

~~Introduction to 2-fold branched Covers~~

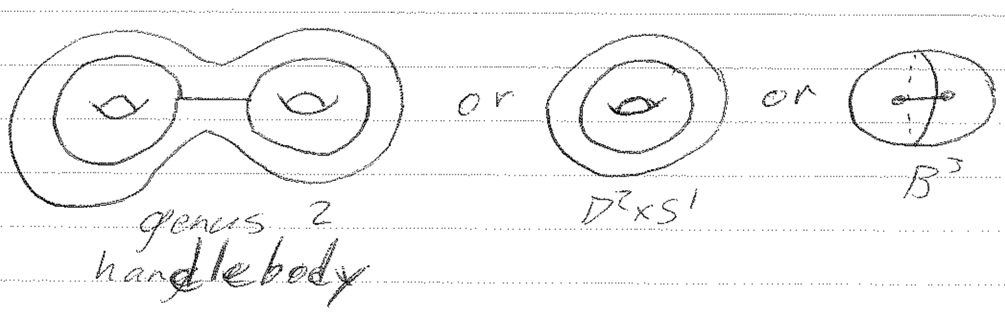
~~Def~~

Def: An n-strand *trivial tangle* is ~~an~~ a proper embedding of $\bigcup_{i=1}^{n} [0,1]$ into B^3 ambient isotopic to



Def: A *handlebody* is a compact 3-manifold homeomorphic to the closed regular nbh of a finite graph embedded in $\mathbb{R}^3$.

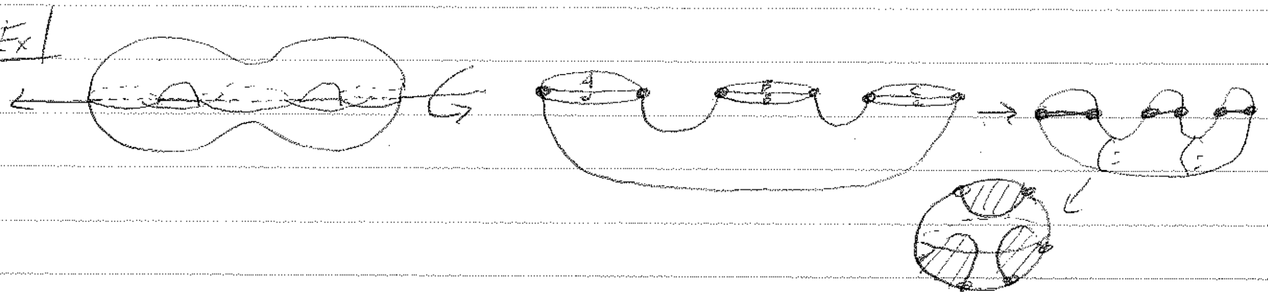
Ex/



Def: A 2-fold branched cover ...

Ex/ The genus n handlebody 2-fold branched covers the n+1 strand trivial tangle

Ex/



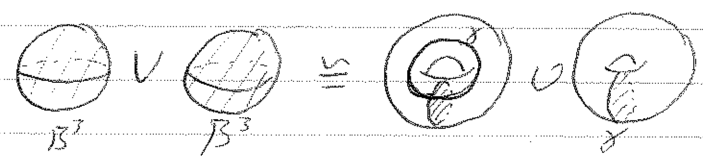
Def A Heegaard splitting of a closed 3-manifold M is a triple (H_1, H_2, f) s.t. H_1 and H_2 are genus g handlebodies, f is a orientation reversing homeomorphism $f: \partial H_1 \to \partial H_2$ s.t. $M \cong H_1 \cup H_2 / \sim$ where $x \sim y$ if $x \in \partial H_1$ and $y \in \partial H_2$ and $f(x) = y$.

Ex

$L(p,q)$ lens space

$$L(p,q) \cong D^2 \times S^1 \cup D^2 \times S^1 / \sim$$

Ex

$S^3 \cong$ 

Def A bridge splitting of a knot K in S^3 is a triple (T_1, T_2, f) s.t. T_1 and T_2 are trivial tangles and f is an orientation reversing homeomorphism $f: \partial T_1 \to \partial T_2$ s.t. $(S^3, K) \cong T_1 \cup T_2 / \sim$ where $x \sim y$ if $x \in \partial T_1$ & $y \in \partial T_2$ and $f(x) = y$

Ex