MATH 555: INTRODUCTION TO 3-MANIFOLDS, HOMEWORK 3

THE TORUS IS PRIME

Due Thursday, 3/9

Problems (to turn in).

(1) In this problem you will prove that the torus surface $T = S^1 \times S^1$ is prime. For the purposes of this problem you will view the torus as a quotient of the square disk, D^2, drawn bellow, according to the standard gluing conventions. Note that a' and a'' are identified to a single simple closed curve, a in T. Similarly, b' and b'' are identified to a single simple closed curve, b, in T.

Assume that γ is an essential separating simple closed curve in T.

i. Show that that γ can be isotoped to intersect both a and b transversely. (Hint: use a theorem from class)

ii. Show that γ can not be isotoped to be disjoint from $a \cup b$.

iii. Show that $a \cap \gamma$ is an even number of points and that $b \cap \gamma$ is an even number of points. (Hint: use that fact that γ is separating.)

iv. Show that, after repeated isotopies, we can assume that no arc of $\gamma \cap D^2$ has both endpoints on a single edge of ∂D^2.

v. By part iv. there are only 6 possible edge types in $\gamma \cap D^2$ according to which edges of ∂D^2 contains their boundary points. Find additional simplifying isotopies of γ to show that we can assume that there are at most 3 edge types realized in $\gamma \cap D^2$.

vi. Use part v. and part iii. to conclude that γ consists of two connected components.

vii. Use part vi. and part ii. to derive a contradiction to the existence of γ and conclude that T is prime.