MATH 550, HOMEWORK 2

BASIS, ORDER TOPOLOGY, SUBSPACE TOPOLOGY, PRODUCT TOPOLOGY AND CLOSED SETS

Due end of day, Thursday, Sept. 21th Note that problems marked with a Q have appeared on past comprehensive exams.

Reading. Read §14, §15, §16 and §17 of Munkres.
Problems.

(1) Munkres §16 exercise 4.
(2) Munkres §16 exercise 9.
(3) Given a set X, prove that $\mathbb{B} = \{\{x\} | x \in X\}$ is a basis for the discrete topology on X.
(4) Let $\mathbb{R}^2$ have the order topology induced by the dictionary order on $\mathbb{R}^2$. Let $S^1 \subset \mathbb{R}^2$ be the standard unit circle. What is the better known name for the subspace topology induced on S^1? Prove your answer.
(5) Q: Assume X and Y are topological spaces and that $A \subset X$ and $B \subset Y$. Let $X \times Y$ have the product topology. Prove that $\overline{A \times B} = \overline{A} \times \overline{B}$.
(6) Q: Are the following true or false. Give a proof or counter example.
    (a) $\overline{int(A)} \subset int(\overline{A})$.
    (b) $int(\overline{A}) \subset \overline{int(A)}$.
    (c) $\overline{A_1 \cup A_2 \cup A_3 \cup ...} = \overline{A_1} \cup \overline{A_2} \cup \overline{A_3} \cup ...$
    (d) $\overline{A - B} = \overline{A} - \overline{B}$.