

MATH 550, HOMEWORK 1

DEFINITION OF TOPOLOGY AND OPEN AND CLOSED SETS

Due end of day, Thursday, Sept. 7th Note that problems marked with a Q have appeared on past comprehensive exams.

Reading. Read §12 and §13 of Munkres.

Problems.

- (1) Munkres §13 exercise 6.
- (2) Munkres §13 exercise 8. (You may use standard facts about the real and rational numbers.)
- (3) Q: Let (X, τ) be a topological space and let $D \subset X$. Prove that $\overline{D} = X$ if and only if D has non-trivial intersection with every element of τ .
- (4) Q: Let τ denote the collection of the empty set and all subsets of \mathbb{R} that contain the element 0.
 - (a) Prove τ is a topology on \mathbb{R} .
 - (b) Find the interior of \mathbb{Q} as a subset of (\mathbb{R}, τ) .
 - (c) Find the interior of $(1, 5)$ as a subset of (\mathbb{R}, τ) .
 - (d) Find the closure of $[-1, 2]$ as a subset of (\mathbb{R}, τ) .
 - (e) Find the closure of $(1, 5)$ as a subset of (\mathbb{R}, τ) .